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GENERAL

PREFACE.

THIS volume, which is issued as Part III. of an Elementary Text-Book of Physics, deals with the elements of *Geometrical Optics.*

In accordance with the plan of the Text-Book, the treatment is of a strictly elementary character, and deals only with the fundamental groundwork of the subject. The usual geometrical methods are adopted, but the physical and experimental aspects of the subject are kept well in view throughout the book.

The experiments given in the text are of a very simple character, and can, in most cases, be carried out with comparatively simple apparatus. The line and point method adopted in some of the experiments in refraction will be found convenient for class work, and the daylight methods given for some of the more important experiments with mirrors and lenses will be found useful where a dark room is not conveniently accessible.

The subject is one in which good diagrams are essential, and for this reason a large number of carefully prepared diagrams have been provided. It is hoped that they may

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be found helpful in making the text clear, and in familiarising the student with the use of diagrams.

Few of the Articles present any special difficulty, so that the book may be read through in a straight-forward manner. The discerning reader will, however, understand that the Introductory Chapter should be read last as well as first.

This Part is complete in itself, but it assumes an elementary knowledge of Wave Motion, as expounded in Part II.

January, 1909.

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LIGHT.

CHAPTER I.

INTRODUCTORY.

1. **The Wave Theory of Light.** — Experimental evidence of a character which we cannot here consider has conclusively established the wave theory of light.

This theory states that light is transverse wave motion in the ether. The vibratory motion of the molecules of a source of light sets up transverse wave motion in the ether. This motion is propagated through the ether with a definite velocity, and when incident on the retina of the eye produces the sensation of sight.

The velocity of propagation of light in ether is found to be about 186,300 miles per second, or nearly 300,000,000 metres per second. This second value is more conveniently written as 3×10^8 metres per second, or 3×10^{10} centimetres per second.

2. **Primary Sources of Light.**—The molecules of every body not absolutely devoid of heat are in vibratory motion, and by their motion are capable of setting up transverse wave motion in the ether. This transverse wave motion in the ether is, in general, known as **radiation.** Although every body is thus a source of radiation, every body is not a source of light, for the incidence of radiation on the retina produces the sensation of sight only when the vibration frequency at the source

is within certain limits. These limits extend from about 400 billion to about 800 billion per second, and, taking the velocity of light in ether to be 3×10^{10} centimetres per second, the corresponding limits of wave length for light waves are ·000075 cm. and ·000038 cm. Light is thus radiation within the limits of wave length specified above, and when incident upon the retina of the eye it produces the sensation of sight.

It is found that a source of light in the sense here defined must be a body at a temperature higher than a red heat, and the primary sources of light with which we are most familiar are usually white hot or **incandescent** bodies. The sun and stars are masses of incandescent matter; a luminous flame is made up of incandescent gases and particles of solid matter; the mantle of a Welsbach light is heated to bright incandescence by the gas flame; the source of light in the electric arc is the incandescent matter in the arc and at the terminals between which it is formed; and the filament of an electric glow lamp is heated to incandescence by the current of electricity passing through it.

There are, however, sources of light other than these high temperature sources. The light emitted by the glow-worm and the fire-fly is not due to incandescence of the source; the light emitted under certain conditions by decaying vegetable matter is also of low temperature origin; and the light given out by certain substances, such as **Balmain's paint,** after exposure to sunlight, or other intense light, is another example of the emission of light from a low temperature source.

3. **Secondary Sources of Light.**—Any body which is brightly illuminated by light from a *self luminous* body or *primary source* of light becomes itself indirectly a *secondary source* of light. The light incident on the body is scattered at its surface, and this scattered light is diffused in all directions from every point on the surface of the body, so that the body becomes a secondary source of light initially derived from a

primary source. Thus, practically all the objects that we see around us in daylight are secondary sources of light derived initially from the sun; the sunlight falling on any object is scattered at the surface of the object, and this scattered light diffused in every direction from every point on the body reaches the eye and renders the object visible. In a room at night where there is no primary source of light none of the objects in it is visible, but directly a "light" is introduced they at once become visible as secondary sources of light. The moon and the planets are not primary sources of light, but secondary sources of the light from the sun by which they are illuminated.

It is important to notice that when light enters the eye and produces the sensation of sight the object seen is the source of the light entering the eye. It is also important to notice that an object can become visible as a secondary source of light only if its surface is capable of scattering or diffusing the incident light. A good reflecting surface, for example, is itself practically invisible; light coming from any point in front of the surface is reflected regularly, so that it appears, after reflexion, to come from a corresponding point behind the surface and *not from any point on the surface.* On the other hand, when light is incident on a rougher surface capable of scattering the light, the scattered light makes, as it were, a fresh start at the surface and diverges in all directions *from every point on the surface.*

4. **Colour.**—Just as the ear distinguishes between sound waves of different wave length by the sensation of pitch, so the eye distinguishes between light waves of different wave length by the sensation of **colour**. Light of the longest wave length produces a red colour sensation, and for shorter wave lengths, *taken individually,* the colour sensation changes through different shades of red, orange, yellow, green, blue, and violet as the wave length shortens to its lowest value for light.

The light waves from any source are very rarely of one particular wave length. They usually include waves of many different wave lengths, so that the colour sensation produced by the light from any given source is usually of a composite character, determined by the combination of the simple colour sensations corresponding to the constituent waves taken individually. Thus, if the light from any source contains, as constituents, waves of wave length corresponding individually to certain shades of red, orange, yellow, and blue the colour sensations produced by the incidence of this light on the retina would be that which results from the combination of these different shades of colour. In this way very complex colour sensations may be excited.

The light from the sun contains waves of *all* wave lengths which affect the retina, and their relative intensity is such that the colour sensation produced by the incidence of sunlight on the retina is that of **whiteness**, and sunlight is, therefore, said to be **white light**. In fact, the light from many incandescent or **white-hot** sources is approximately white. Thus, the light from a Welsbach light is white slightly tinged with green, due to excess of the green constituent; and light from ordinary gas, lamp, and candle flames usually has a reddish-yellow tinge, due to the weakness of the blue constituents. The light from the electric arc is almost as white as sunlight; but the light from electric glow lamps is, like that from ordinary luminous flames, defective in blue. **Blackness** is merely the entire absence of light sensation, such as is experienced in complete darkness.

Experiment 1.—Take a Welsbach light and a candle flame and hold a pencil or ruler in front of a sheet of *white* paper so that the two shadows cast on the paper by the two sources of light are close together. Note the difference in the colour of the paper in each shadow and outside the shadows.

The paper in shadow from the candle flame is illuminated by the gas flame, the part in shadow from the gas flame is illuminated by the candle flame, and the rest of the surface is illuminated by both flames.

The colour sensation produced by the light incident on the eye from any source is referred to its origin, and taken as the colour of the source. Thus, the sun is said to be white because the light from it produces the sensation of whiteness. If the source is an illuminated object, acting as a secondary source, then the colour sensation produced is ascribed to the source as the colour of the object seen. The colour of any object acting as a secondary source depends ultimately upon the light diffused from its surface, but primarily upon the light incident on its surface and on the nature of the surface. The incident light is not only scattered at the surface, but may also undergo absorption at the surface—that is, some of the wave motion energy of the incident light may be communicated to the surface molecules so as to increase the molecular vibration energy or heat of the surface layers. The light diffused from the surface will, therefore, differ from the incident light by the absence of the light absorbed at the surface, and thus the colour of the object as seen is really the colour of the incident light deprived of the constituents absorbed at the surface. Thus, paper which appears white in sunlight appears white because it does not absorb any of the incident light, and, therefore, the light diffused by it is unaltered white sunlight. This paper would evidently always appear to be of the colour of the source by which it is seen; this explains why *white* paper must be used in the experiment described above. Also paper, or any other object, which appears *red* in sunlight probably absorbs practically all the yellow, green, and blue constituents, so that the diffused light is wanting in these constituents, and, therefore, gives a sensation of which red is the main constituent.

Objects seen as secondary sources by light derived from the *same* primary source may thus be of many different colours, for although the same light is incident on each object, the surfaces of the objects may differ, so that the light absorbed at different surfaces, or even at different points on the same surface, is quite different, and, therefore, the residual diffused light from these surfaces may give rise to many different colour sensations.

5. **The Ether.**—The ether is the medium which is supposed to fill all space and permeate all matter. Space which is a *vacuum* as far as matter is concerned is filled with ether, and in matter itself ether fills up the spaces between the molecules in much the same way as a jelly might fill up the spaces between an assemblage of marbles set close together in the jelly without touching each other. The ether is thus supposed to be a continuous medium extending throughout space and matter.

The properties of the ether are still, to some extent, a matter of speculation. It is not perceptible by any of our senses, and its properties can, therefore, be deduced only from observation of the phenomena that are supposed to take place in it. The phenomena of light and electricity are ether phenomena, and all the knowledge which we possess of the properties of ether is derived from the study of these subjects.

It must be remembered that the ether is always the medium through which light is transmitted. When light travels through space empty of matter it is carried by the ether which fills the space. When it travels through material media it is carried by the ether which permeates the matter, and holds the molecules of matter, as it were, in suspension. The properties of the ether as a medium for the propagation of light appears, however, to be affected by the matter with which it is associated or loaded. Thus, the velocity of light in free ether—that is, in ether free from matter—is about 3×10^8 metres per second, while the velocity of light through water—that is, through the ether which permeates water—is about $2{\cdot}25 \times 10^8$ metres per second, and through glass it is about 2×10^8 metres per second.

6. **Opaque and Transparent Media.**—Light is propagated through free ether without sensible loss by absorption in the medium, but in ether associated with matter there may be very great loss of energy due to absorption.* Thus light cannot be transmitted for any appreciable distance through certain material media because the energy of the wave motion is completely absorbed by a very small thickness of the medium. Certain other material media, however, such as air, glass, water, &c., allow light to pass through considerable thicknesses without very great loss by absorption. Substances through which light cannot be propagated for any appreciable distance on account of absorption are said to be **opaque** to light, and substances through which light can be transmitted for considerable distances without undue loss by absorption are said to be **transparent.** It must be noted, however, that the difference between opaque and transparent substances is simply one of degree. In an opaque substance light is absorbed by a very small thickness of the substance; in a transparent substance light is absorbed only by a comparatively great thickness, and so may pass through ordinary thicknesses without much loss of intensity. Thus a thin sheet of metal is said to be opaque to light, but experiment shows that light can pass through a thin film of metal. Gold leaf, for example, if placed between two sheets of glass and held up in front of a bright light is seen to be transparent. Similarly, a sheet of horn an eighth of an inch thick is quite opaque for ordinary sources of light, but thin sheets, such as were used in horn lanterns, are fairly transparent. Familiar examples of transparent substances are air and most gases, water and many liquids, glass, ice, rock-crystal, mica, rock-salt, Iceland spar, diamond, and many other crystalline substances.

Some transparent substances are of such a structure that they scatter the light as it passes through them. These substances, such as porcelain and opal glass, are said to be **translucent**. Similarly, ground glass appears translucent on account of the scattering of the light at the ground or roughened surface of the glass.

---

* The energy absorbed is converted into heat *in the material medium.*

7. **Colour of Transparent Substances.**—When light, consisting of waves of many different wave lengths, passes through a transparent substance it sometimes happens that waves of different wave length are **unequally** absorbed, so that the composition of the emergent light is quite different from that of the light entering the substance. In this case the transparent substance, when seen by the light transmitted through it, will be **coloured**, the colour being determined by the composition of the emergent light. Thus red glass appears red when seen in daylight by transmitted light, because the sunlight in passing through it is so changed in composition by the unequal absorption of its constituents that when incident upon the eye it gives a colour sensation of which red is the main constituent.

If a colourless transparent substance is reduced to powder it appears, when seen by white light, to be of a pure white colour, because the light incident on the broken surface of the powder is scattered and diffused in all directions from the surface after penetrating only a very thin layer of surface particles. The light thus diffused from the surface has, therefore, suffered practically no loss by absorption, and does not differ appreciably from the white light incident on the powder.

Similarly, if a coloured transparent substance, such as coloured glass, is reduced to powder it appears when seen by white light to be of a white colour only slightly tinged with the colour of the glass. This is because the light diffused from the surface of the powder has passed through a very thin layer of surface particles, and the effect of the absorption in this thin layer is sufficient to give only a slight tinge of colour to the powder.

CHAPTER II.

RECTILINEAR PROPAGATION OF LIGHT.

8. **Rectilinear Propagation of Light is Consistent with the Wave Theory.**—Light waves from a luminous point as source spread out into the surrounding ether as a spherical wave. It is found, however, that the disturbance which reaches any point in the ether travels only along the line joining the source to that point, and is propagated on from this point only along this line produced. Thus, in Fig. 1, if the luminous point be at O, the light which reaches any point P travels to it only along the straight line O P, and is propagated on into the ether only along PQ the continuation of OP. That is, the disturbance which travels along the line OPQ from the source does not extend laterally from this line into the medium and cannot, therefore, affect the ether at any point close to the line as at R or S. Hence, if an opaque obstacle be placed across this line at any point the disturbance along the line does not travel beyond the obstacle. Further, if the obstacle is so placed, as at M, that the line OPQ just grazes the edge of it, the disturbance travelling along MQ does not spread into the medium behind the obstacle and cannot reach any point on the same side of the line as the point S. It follows from this that, if an opaque screen is placed anywhere in the path of the light from O, no light reaches any point behind the screen on a line drawn through the screen. That is, the space behind the screen, within lines drawn from O through the boundary of the screen, is in shadow.

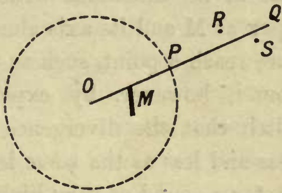

Fig. 1.

The fact that the light disturbance reaching any point in the ether from a luminous point as source, travels only along the straight line from the source through the point is conveniently expressed by saying that the propagation of light from a point source is rectilinear, or along straight lines from the source. It can be shown that approximately rectilinear propagation is consistent with the wave theory if the wave length is very small, but that it cannot be strictly true even for the smallest wave lengths.

Thus, if O were a source of sound in a material medium, and a sound proof obstacle were placed as at M, it would be found that the sound could be heard distinctly at points behind the obstacle on the shadow side of MQ. That is, the disturbance from the point M is propagated not only along MQ, but spreads out in all directions within a cone of small angle, having its apex at M and its axis along MQ. The disturbance can therefore reach a point, such as S on the shadow side of MQ. It is found, however, by experimenting with sounds of different pitch that the divergence from straight line propagation gets less and less as the wave length of the sound decreases. That is, for a sound of very high pitch, although the sound can still be heard within the shadow region, it can be heard only at points fairly close to the line MQ.

Now the wave length of light is so very small that the divergence from straight line propagation in its propagation through ether as transverse wave motion, is almost inappreciably small. This divergence, however, exists, and can be detected by experiment. Thus, it is found that the shadow of a straight edge cast on a screen by a luminous point of white light is not sharply defined, but is fringed **on the shadow side** by coloured lines or fringes due to the divergence from straight line propagation of the constituent waves of different wave lengths which come from the source. These constituent waves being of different wave lengths diverge to different extents, and, as they correspond to different colour sensations, they give rise to the coloured fringes seen on the screen. This divergence from strict rectilinear propagation is known as the **diffraction** of light.

9. **Rays of Light.**—The rectilinear propagation of light makes it permissible to consider the light diverging from any point source as an assemblage of lines of wave disturbance travelling along straight lines diverging in all directions from the source. These lines of wave disturbance are called **rays** of light, and a ray of light may be defined as the wave disturbance travelling from a luminous point source, or from a point on any luminous source, along a straight line in any direction from the source.

This conception of rays of light greatly simplifies the study of the simpler phenomena of light. The rays can be represented by straight lines, and simple geometrical methods can, therefore, be applied in the treatment of the subject. This geometrical treatment of **Light** or **Optics** is known as **Geometrical Optics.**

It must be remembered, however, that rectilinear propagation of light is only approximately true, and that many of the phenomena of light, such as **Interference** and **Diffraction**, are, therefore, beyond the scope of the methods of geometrical optics. A full and complete exposition of the phenomena of light is, however, given by the science of **Physical Optics**, which is based on the wave theory of light. In this book we are concerned only with the elements of Geometrical Optics.

As defined above, a ray is the wave disturbance travelling along a **line**, and cannot, therefore, be realised experimentally. In practice we have to deal with the disturbance travelling within a very narrow cone or cylinder, and including a conical or cylindrical bundle of rays known as a **beam** or a **pencil** of light. The term **pencil** is generally used when the beam is very narrow, and approximates as nearly as possible to a line.

A beam or pencil of light may be **divergent, convergent**, or parallel—that is, the component rays may diverge from, or converge to a point, or run parallel, as shown at (*a*) (*b*), and (*c*) in Fig. 2.

A convergent or divergent beam or pencil has the form of a cone of small angle, while a parallel beam is a cylinder of small cross-section. The **axis** of a beam, FA, is the central ray passing along the geometrical axis of the figure of the beam, and the point F, from or to which the rays of the beam diverge or converge, is called its **focus**. The focus of a parallel beam is at infinity.

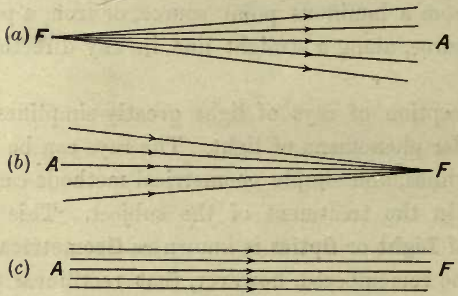

Fig. 2.

Light coming from any point on a real source of light is necessarily divergent. A pencil of light *from a point* on a very distant source, such as the sun, is, however, practically parallel although really slightly divergent. It will also be found later that a divergent pencil may become parallel or convergent after reflection at a spherical mirror or refraction through a lens.

10. **Illustrations of the Rectilinear Propagation of Light.**—The medium through which light is propagated is always the ether. It has, however, been explained in Art. 5 that the properties of the ether as a medium for the propagation of light are different in different material media. Hence, when light is travelling through the ether in any material medium it is usual to speak of this medium as the medium of propagation.

Thus when light is travelling through the ether in glass we speak of glass as the medium through which the light is propagated.

In any given homogeneous medium light travels in straight lines, but when light passes from one medium to another it is, in general, bent out of the rectilinear path.

Experiment 2.—Let two screens be pierced, each with a small pin-hole, and then hold one in front of the other, as shown in Fig. 3, so that the two holes and the glowing tip of the wick in a candle flame (or, in daylight, the intersection of two crossed wires or hairs fixed in a frame) are in the same straight line.

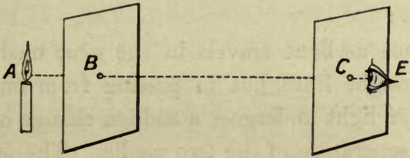

Fig. 3.

Now place the eye behind the screens so that the glowing tip of the wick can be seen, and note that the slightest displacement of either screen in its own plane, or, in fact, any displacement of A, B, C, or E, which interrupts the straight line path from A to E, renders the object at A invisible.

Experiment 3.—Arrange a scale, as shown in Fig. 4, on the bottom of a trough, and note that when the scale is viewed from a point E over the edge of the trough at C, the point A at which the line of sight ECA cuts the scale is such that ECA is a *straight line.*

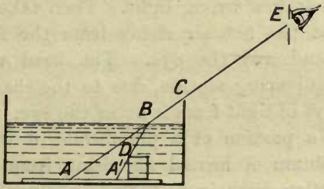

Fig. 4.

The point E may be fixed by looking through a hole in a screen.

Now fill the trough half full of water. A different point, A′, on the scale will now be seen from E over the edge of the trough.

Place a metal cube or prism on the scale in such a position that one of its upper edges is parallel to the edge at C, and is so placed that it appears *under water* in the line of sight between E and A′. This fixes a point, D, on the portion of the line of sight which lies in the water. Fix also the point B where the line of sight cuts the surface of the water by means of a scale fixed horizontally at the surface.

It will be found that the lines A′DB and BCE are both straight, but that they are not in the same straight line; there is a sudden change of direction at the point B where the light passes from water to air.

Hence, so long as light travels in the same medium the rays of light are straight lines, but in passing from one medium to another a ray of light undergoes a sudden change of direction at the surface of separation of the two media. The magnitude and direction of this change are dealt with more fully in the chapter on *Refraction.*

This sudden change of direction is evidently due to the abrupt passage of the ray from one uniform medium to another. Hence, if light is travelling in a medium which is not uniform or homogeneous throughout, a ray may suffer gradual and irregular changes of direction.

Experiment 4.—Take a small bright bead, mounted on a black stem, and place it in a strong light. Then take a Bunsen flame and place it so that the hot air rising from the flame comes directly between the bead and the eye. The bead will appear to have a flickering or quivering motion, due to the changes in the direction in which the rays of light from it enter the eye.

Similarly, if a portion of a plumb-line of white cord is viewed through the column of heated air rising from a flame, the line is seen as an irregular, slightly wavy line, continually changing in form, and not as the straight vertical line it really is.

This irregular and fluctuating change of direction in the rays of light caused by want of homogeneity in the medium through which the rays are travelling explains the quivering appearance of objects seen through the heated air lying over strongly heated ground or sand on a warm summer day.

It should be noticed in connection with these experiments that the eye takes no cognisance of change of direction of a ray or pencil of light; any point on a source of light is seen in the direction which the axis of the pencil of light which enters the eye from the point has *on entering the eye*, and its apparent position on this axis is at the geometrical focus of the rays entering the eye. For example, if light starting from a point O, Fig. 5, be bent as indicated in the figure, the point O appears to the eye to be at O′. This point, O′, is the apparent or **virtual focus** of the pencil of light entering the eye; it is called a **virtual** focus because the rays entering the eye do not *really* come from it, but only appear to the eye to do so.

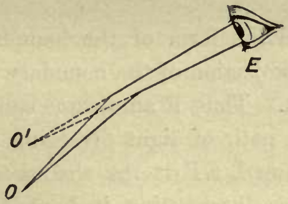

Fig. 5.

11. **Shadows.**—The formation of light shadows is a direct consequence of the rectilinear propagation of light. If an opaque object is placed in the path of a beam of light, the space behind the object and within the surface formed by the rays which just graze the boundary edge of the object, is in shadow.

The characteristics of the shadow formed in any particular case depend upon the relative sizes of the source of light and the opaque object.

If the source of light be a luminous point, or a very small source compared with the size of the opaque object, the shadow is of a very simple character. The cone of light from the point which is incident on the surface of the opaque body is stopped, and the space beyond, enclosed by the geometrical continuation of this cone, is screened from the rays diverging from the luminous point, and is therefore in complete shadow. This cone, having its apex at the luminous point and defining the limits of the shadow region behind the opaque body, is called the **shadow cone.** If a screen is placed so that the shadow of the obstacle is thrown on it, the shadow area on the screen is a section of this cone of shadow, and the form of the boundary of the shadow area is evidently a projection of the boundary edge of the opaque object on the screen. Thus, if an opaque sphere be placed as at A B, Fig. 6, in the path of light diverging from a very small source at L, the cone L E F is the shadow cone, and its trace, C D, on any surface intersecting it beyond A B outlines the **shadow** cast on this surface. In this case the shadow cast on any plane surface is a section of a circular cone, and its boundary, if complete, is therefore a circle or an ellipse. The extent of the shadow area on the screen evidently increases as the distance between the obstacle and the screen increases. Thus, in Fig. 7, if A B represents any linear dimension of the boundary* of the

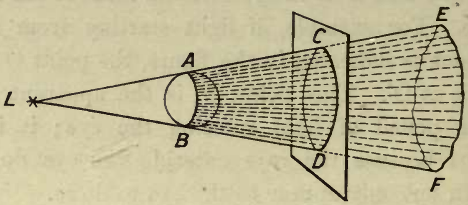

Fig. 6.

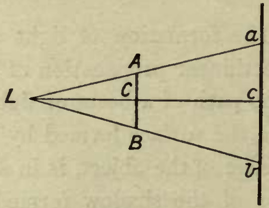

Fig. 7.

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* Defined by the tangential rays from L.

opaque object, and ab the corresponding dimension of the shadow area on the screen, the triangles LAB and Lab are similar, and, therefore, $\frac{\text{AB}}{ab} = \frac{\text{LA}}{\text{L}a} = \frac{\text{LC}}{\text{L}c}$. The plane of the screen is here supposed to be parallel to the line AB.

Numerical Example.—A circular disc, 6 inches in diameter, is suspended in a vertical plane parallel to a vertical screen, 4 feet from it. A very small source of light is placed, so that it casts a shadow of the disc on the screen, at a distance of 6 feet from the disc, on a line perpendicular to the screen through the highest point on the disc. Find the shape and dimensions of the shaded area on the screen.

The shadow on the screen is a section of the shadow cone* *parallel to the circular section at the disc*, and its boundary is therefore a *circle*. Also, we have—

$$\frac{\text{The diameter of the shadow}}{\text{The diameter of the disc}} = \frac{\text{Distance of screen from source}}{\text{Distance of disc from source}}.$$

That is,

$$\frac{\text{The diameter of the shadow in inches}}{\text{6 inches}} = \frac{10}{4} = \frac{5}{2}.$$

$\therefore$ The diameter of the shadow = 15 inches. The shadow on the screen is, therefore, circular in form and 15 inches in diameter.

When the source of light is not a point, but of finite size, the shadow must obviously be of a compound character, for a shadow of the obstacle is formed by every luminous point on the source, and the shadow formed by the source as a whole is therefore compounded of the shadows formed by the individual points on it. The region in this compound shadow, which is common to the shadows formed by *all* the points of the source, is a region of **total shadow** and is called the **umbra.** The region which is common only to the shadows cast by *some* of the points on the source is a region of **partial shadow** known as the **penumbra.**

Experiment 5.—Place a bright light behind a small hole in a screen, and arrange that the shadow of a ball is thrown by this point source of light on a screen. Notice (a) that the shadow area on the screen is uniformly dark, (b) that its edges are clearly defined, and (c)

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* The *right section* of this cone is not a circle.

that it is circular or elliptical in form (according as the screen is parallel to or inclined to the circle of contact of the shadow cone with the ball), and increases in area as the distance of the screen from the ball is increased.

Make measurements of AB, *ab*, LC, and Lc (Fig. 7), and verify the relation given above between these distances.

Experiment 6.—Take a large flat gas flame as a source of light. Place the flame edgeways in front of a screen, so that it casts the shadow of a vertical rod, placed in the plane of the flame, on the screen. Examine the shadow, and note that it is similar to that examined in the last experiment. It will be seen that this is so, because, in any horizontal section of the flame, rod, and screen, the shadow of the width of the rod is formed by the width of the front edge of the flame, which is practically a luminous point.

Now place the flame with its flat surface parallel to the screen, and let it throw the shadow of any cylindrical object on the screen. Examine the shadow area on the screen, and notice the central band of total shadow (the **umbra**), bordered on each side by an ill-defined band of partial shadow (the **penumbra**). Notice, also, that the absolute and relative widths of these bands vary considerably as the distance between the obstacle and the screen is changed.

The peculiarities of a compound shadow under different conditions are illustrated by Figs. 8, 9, and 10. In these figures LL′ and AA′ represent respectively the source of light and the opaque body. Lines from L and L′, the *extreme* points (in the plane of the figure) on the source of light, are drawn through A and A′ so as to define the simple shadow cones, LUP and L′U′P′, for the two *extreme points* of the source. The shadow cone for any other point on the source lies *within* the limits defined by these four lines which, therefore, define the limits, in the plane of the paper, of the shadow formed by the source as a whole. It will be seen that the limits of the umbra and penumbra of the compound shadow are defined by the cones having their apices at the points O and O′ respectively. The region of total shadow, common to the simple shadows cast by all the points on the source, evidently lies within the cone diverging from O, and the region of partial shadow lies between this cone and the double cone diverging from O′, and forming the outer boundary of the compound shadow.

The single cone LL′UU′, which encloses LL′ and AA′, and has its apex at O, is, therefore, the cone of **total shadow** or **the cone of the umbra**, and the portion of it screened from the light, behind AA′, is called the **umbra**.

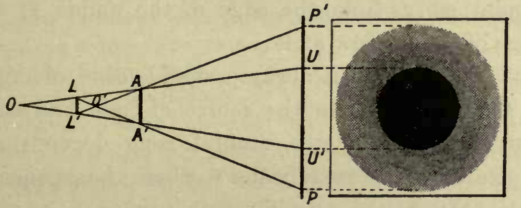

Fig. 8.

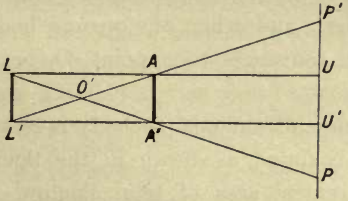

Fig. 9.

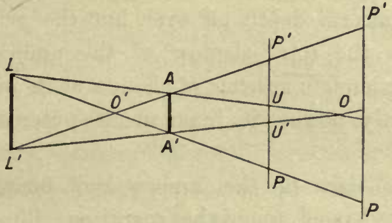

Fig. 10.

The double cone LL′O′PP′, enclosing LL′ and AA′, and having its apex at O′ between the source of light and the opaque body, is the cone of **partial shadow** or **the cone of the penumbra**, and the portion of it beyond AA′ and surrounding the umbra is called the **penumbra**.

Any point in the penumbra is in shadow only from a portion of the source, so that the depth of the shadow at any point depends upon the extent of the source from which the point is screened. Thus, in Fig. 8, any point between U and P′ is in shadow from the portion of the source below the line of sight through A, and the depth of the shadow evidently decreases as the point moves from the edge of the umbra at U to the outer edge of the shadow at P′.

The penumbral cone is always a double cone diverging from a point, O′, lying between the source of light and the opaque body, but the form of the umbral cone depends upon the relative size of these two bodies; when the opaque body is the greater, Fig. 8, the cone diverges from a point, O, behind the source; when they are equal, Fig. 9, the cone takes the form of a cylinder; and when the opaque body is the smaller, Fig. 10, the cone converges to a point, O, beyond the opaque body.

When the shadow of the opaque body is cast upon a suitably placed screen, it is found, as shown in the figures given above, to consist of a central area of total shadow, the umbra, surrounded by a ring of partial shadow, the penumbra. The umbra is of uniform depth all over, but the penumbra passes gradually from the total shadow of the umbra at its inner boundary to a complete absence of shadow at its outer boundary, and, consequently, neither its inner nor its outer edge is sharply defined.

The relative sizes of the umbra and penumbra in any particular case depend upon the conditions illustrated in the figures and upon the position of the screen. In the case illustrated by Fig. 10 the umbral cone is not continued beyond the apex at O; inside the geometrical continuation of this cone beyond O a ring of the source of light would be visible round the boundary of the opaque object, and the width of this ring increases as the distance beyond the apex at O increases.

Experiment 7.—Get a piece of sheet zinc about 6 inches square and cut a circular hole about an inch in diameter in the centre of it. Fix this piece of zinc in front of, and close to, the flat surface of a large flat gas flame, so that a circular portion of the brightest part of the flame shows through the hole. This gives a circular luminous surface as a source of light.

Now suspend a ball in front of the source of light so that a shadow of the ball is thrown on a screen placed parallel to the source. The ball should be placed with its centre on the perpendicular from the centre of the source on to the screen.

Study the shadow thrown on the screen at various distances from the ball, (*a*) when the diameter of the ball is greater, and (*b*) when it is less than that of the source of light. In the latter case (*b*), trace the umbra to its apex and notice the character of the shadow within the geometrical continuation of the umbral cone at different distances beyond the apex. This experiment should be conducted in a dark room, and the screen should be protected from diffused light from the flame. A sheet zinc chimney of square section, with a circular hole cut in one side, is, for this purpose, more satisfactory than the simple square of sheet zinc described above.

Experiment 8.—Hold a ruler or ball so that the sun casts its shadow on a wall or on the ground or on any convenient screen. Note the general character of the shadow and examine it carefully, (*a*) when the ruler is *close to* the screen, and (*b*) as the distance of the ruler from the screen increases.

It will be seen that the shadow is of the type shown in Fig. 10, and that, as the distance between the ruler and the screen increases, the penumbra (at first very narrow) gets wider, while the umbra gets narrower, and finally disappears at the apex of the umbral cone.

12. **Eclipses.**—As the moon revolves round the earth it sometimes comes between the earth and the sun, as in Fig. 11, so that its shadow is cast on the surface of the earth. When this happens an observer within the shadow of the moon on the earth's surface sees the sun totally or partially eclipsed by the moon. This is a **solar eclipse.**

At other times the position of the moon is such, Fig. 12, that the earth lies directly between it and the sun, and a shadow of the earth is, therefore, cast on the surface of the moon. When this happens an observer on the earth's surface sees the bright surface of the moon wholly or partially eclipsed by the shadow of the earth thrown on its surface. This is a **lunar eclipse.**

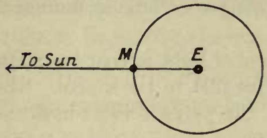

Fig. 11.

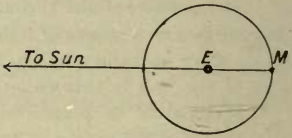

Fig. 12.

A **solar eclipse** can evidently take place only at new moon, and would take place regularly every month at this time were it not that the plane of the moon's orbit round the earth makes a small angle with the plane of the earth's orbit round the sun. For this reason the moon generally passes the new moon position without coming directly between the sun and the earth, and its shadow, thrown into space, does not fall on the earth's surface. Occasionally, however, the position of the moon at new moon is such that its shadow falls somewhere on the earth's surface.

It will be evident on consideration that an eclipse of the sun can be visible on the earth's surface only at places which lie on the track of the moon's shadow over the surface.

The diameter of the sun is much greater than that of the moon, so that the moon's shadow, as cast by the sun on the earth, is of the type shown in Fig. 10. The relative size of these two bodies and their distances from the earth are such that the surface of the earth lies close to the apex of the umbral cone; usually it is slightly nearer the sun than this point, but occasionally it lies just beyond the apex.

When the umbral cone reaches the earth, the shadow on the earth's surface consists of a comparatively small central umbra and the surrounding penumbra. Any point on the narrow track of the umbra is in complete shadow when the moon's shadow passes over it, so that the eclipse of the sun, as seen from any place on this track, is a **total eclipse.** Similarly the eclipse, as seen from a place on the track of the penumbra, is only a **partial eclipse.**

When the earth is just beyond the reach of the umbral cone, the shadow on the earth's surface consists of a central region, within the geometrical continuation of the umbral cone, which is illuminated by a ring of the sun's disc, and the surrounding penumbra. Hence, for places on the track of the central portion of this shadow, during the eclipse a ring or **annulus** of the sun's disc is seen round the dark moon, and the eclipse is said to be an **annular eclipse.** For places on the track of the penumbra the eclipse is, as explained above, a **partial eclipse.**

A **lunar eclipse** by the shadow of the earth thrown on the moon by the sun can evidently take place only at full moon, and ought to take place regularly at this time every month were it not, as already explained, that the sun, earth, and moon are not generally in the same plane. Here again the opaque body, the earth, is much smaller than the source of light, the sun, so that the shadow cast on the moon is, as in the case of a solar eclipse, of the type shown in Fig. 10. The relative sizes of the sun and the earth, and the relative distances of the sun and moon from the earth, are such that the cross-section of the umbral cone, at the point where it falls on the moon, is about equal to the moon's disc. Hence, if the earth's shadow passes centrally over the moon, the umbra just covers the disc, and the eclipse is seen from the places on the earth's surface at which the moon is visible, as a total eclipse. If the earth's shadow does not pass centrally over the moon's disc, the umbra covers only a segment of the disc, and the eclipse is only partial. The passage of the penumbra of the earth's shadow over the moon's surface does not darken the surface sufficiently to produce an eclipse; it causes, however, a marked diminution of brightness, and the colour of the surface changes to a dull red or copper colour.

13. **The Pin-hole Camera.**—When a sheet of cardboard

with a large pin-hole in the centre is placed, as shown in Fig. 13, between a candle flame and a screen in a dark room, a more or less distinct, but inverted, representation of the candle flame is formed on the screen.

If the cardboard sheet form the front, and the screen the back of a closed box, a representation of any well-lighted object in front of the box is thrown on the screen at the back, and can be seen distinctly from behind, through the screen, if this is made of some transparent material, such as thin paper or ground glass, and is properly shaded from extraneous light. A closed box of this kind, with a pin-hole in the centre of the front and a screen of tracing linen or ground glass fitted into the back, is called a **pin-hole camera.** The sides of the camera should project some distance beyond the screen at the back, so as to shade it as much as possible from external light.

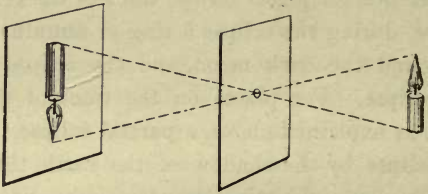

Fig. 13.

Experiment 9.—Take a large tin canister, as shown in Fig. 14, pierce a small hole, O, in the centre of the bottom, and fit a ring of cardboard, CC, carrying a transparent screen, SS, into it about midway between the two ends, as shown in the figure. This gives a simple, but effective, pin-hole camera: an inverted image of objects in front of the pin-hole at O can be seen on the screen by looking in at the open end of the canister.

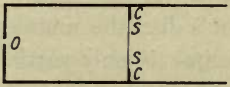

Fig. 14.

The explanation of the action of the pin-hole camera is simple. Let AB (Fig. 15) represent any well-lighted object before the camera, O the hole in front of the camera, and SS the screen. From every point on AB small pencils of rays pass through the pin-hole at O, and form small circular or elliptical spots of light on the screen. Every point on the object is, therefore, represented by a spot on the screen, and these spots, therefore, define a representation or **image** of the **object** on the screen. The image is **inverted** on account of the crossing of the pencils at O, and it should be noticed that it is **inverted**, not in a vertical direction only, but in every direction; that is, if the object were circular in form, every diameter on the object would be **inverted** or reversed in the image. If the spots are large they overlap, and the image is blurred and indistinct. Hence, in order to obtain a well-defined image, the spots must be very small—that is, the pin-hole must be very small, for the size of a spot on the screen, corresponding to any luminous point on the object, evidently depends, for given positions of AB and SS, upon the size of the pin-hole.

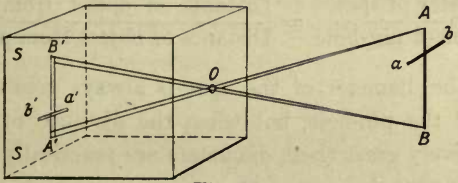

Fig. 15.

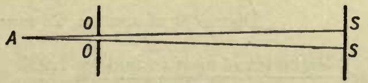

Fig. 16.

The size of the spots for a hole of given size depends also upon the distance of the object from the hole; the greater this distance is the smaller the spots on the screen. Thus, in Fig. 16, let OO represent the diameter of the pin-hole, and SS the diameter of a spot on the screen, as formed by a pencil of rays from a point A on the object. Then, the triangles AOO and ASS being similar, we have

$$\frac{\text{SS}}{\text{OO}} = \frac{\text{AS}}{\text{AO}}.$$

That is,

$$\frac{\text{Diameter of spot}}{\text{Diameter of pin-hole}} = \frac{\text{Distance of object from screen}}{\text{Distance of object from pin-hole}}.$$

Obviously the diameter of the spot is always greater than the diameter of the pin-hole, but when the distance of the object is comparatively great these diameters are practically equal.

Numerical Example.—If the depth of the camera is 25 cms. and the diameter of the pin-hole ·2 mm., find the diameter of the spots making up the image, when the distance of the object from the pin-hole is (*a*) 50 cms., (*b*) 1 metre, (*c*) 10 metres.

Here, from the data given, we have—

$$(a)\quad \frac{\text{Diameter of spot in mm.}}{\cdot 2\text{ mm.}} = \frac{75}{50} = \frac{3}{2},$$

or,

$$\text{Diameter of spot} = \cdot 3\text{ mm.}$$

$$(b)\quad \frac{\text{Diameter of spot in mm.}}{\cdot 2\text{ mm.}} = \frac{125}{100} = \frac{5}{4},$$

or,

$$\text{Diameter of spot} = \cdot 25\text{ mm.}$$

$$(c)\quad \frac{\text{Diameter of spot in mm.}}{\cdot 2\text{ mm.}} = \frac{1{,}025}{1{,}000},$$

or,

$$\text{Diameter of spot} = \cdot 205\text{ mm.}$$

Although a small pin-hole is essential for a good image, the **sharpness** or **definition** of the image really depends upon the distance between points on the object which are represented, on the image, by spots which touch each other, but do not overlap. The image is well defined if the distance is small and ill defined if the distance is great. This distance is readily expressed in terms of the diameter of the pin-hole.

Let P and Q, Fig. 17, be two points on the object which give two contiguous, but distinct, spots, RS and ST, on the screen, through the hole OO. The triangles SPQ and SOO are evidently similar, and, therefore—

$$\frac{\mathrm{PQ}}{\mathrm{OO}} = \frac{\mathrm{SP}}{\mathrm{SO}},$$

or, if the distance PQ be denoted by x; OO, the diameter of the hole, by a; SP, the distance of the object from the screen, by D; and SO, the depth of the camera, by d, then—

$$\frac{x}{a} = \frac{\mathrm{D}}{d}, \text{ or } x = \frac{\mathrm{D}}{d}a.$$

That is, the distance x, which determines the definition of the image, depends upon the distances D, d, and a, and the smaller D and a are, and the larger d is, the better the definition of the image.

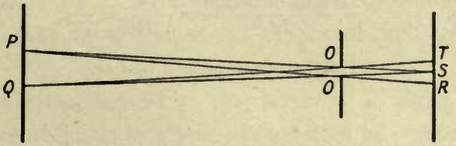

Fig. 17.

Numerical Example.—Thus, if the depth of the camera from front to back is 10 inches and the diameter of the pin-hole is ·01 inch, the value of x for an object 20 feet from the pin-hole is given by $x = \frac{250}{10} \times {\cdot}01$ inch $= {\cdot}25$ inch. That is, points a quarter of an inch apart on the object are represented by separate spots on the screen.

The diameter of the spots in this case is $\frac{250}{240} \times {\cdot}01$ inch $= {\cdot}0104$ inch, or very little larger than the diameter of the pin-hole.

The relative size of the image and object can also be readily determined. In Fig. 15 let AB and A′B′ represent respectively the lengths of the object in the image. The triangles AOB and A′OB′ are similar, and, therefore—

$$\frac{A'B'}{AB} = \frac{OA'}{OA}.$$

Thus, for *linear* dimensions only,

$$\frac{\text{Size of image}}{\text{Size of object}} = \frac{\text{Depth of camera}}{\text{Distance of object from pin-hole}},$$

or,

$$\frac{\text{Size of image}}{\text{Size of object}} = \frac{\text{Distance of image from pin-hole}}{\text{Distance of object from pin-hole}}.$$

Numerical Example.—Thus, if the depth of the camera from pin-hole to screen is 10 inches, and the image of an object 6 feet high and 120 feet from the pin-hole, is thrown on the screen, the height of the image is given in inches by the relation :

$$\frac{\text{Height of image}}{72} = \frac{10}{1{,}440} = \frac{5}{72},$$

or,

$$\text{Height of image} = 5 \text{ inches}.$$

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CHAPTER III.

PHOTOMETRY.

14. **Intensity of Light.**—The intensity of light, considered as wave motion in the ether, is measured at any point by the energy per unit area, transmitted in unit time across the wave front at the point considered. Thus, if a quantity of energy q is transmitted in one second across a small area s at any point on a wave front, then q/s is a measure of the intensity of the light at that point.

It follows from this, as in the case of sound, that the intensity of light from a point source, at different distances from the source, must vary inversely as the square of the distance from the source.

The direct measurement of the intensity of light, as defined above, involves difficulties which cannot here be considered. For practical purposes, however, the intensity of the light from any source may be taken as indicated at any point by the illumination produced on a suitable screen placed at the point considered. The screen should be placed so that the rays of light are directly and not obliquely incident on it, and its surface should be such that practically *all* the incident light is diffused or scattered by it, and suffers no appreciable loss by absorption or regular reflection at the surface. A screen of white unglazed paper forms an efficient screen for this purpose.

15. **Intensity of Surface Illumination.**—The intensity of illumination of a surface is appreciable to the eye by the visual **brightness** of the surface. The eye, however, is quite unable to *measure* the intensity of the illumination of a surface; nor can it even compare the intensities of two unequally illumin-

ated surfaces. It can, however, decide with some accuracy whether two adjacent surfaces are equally or unequally illuminated, and this power is the basis of all practical methods for the comparison of the illuminating powers of different sources of light. It is necessary, therefore, in order to systematize the theory of these practical methods, to define intensity of surface illumination and also the illuminating power of a source of light.

For any uniformly illuminated surface, the intensity of the surface illumination is measured by the quantity of light per unit area incident in one second on the surface. Thus, if Q denote the quantity of light incident per second on a uniformly illuminated area S, then Q/S measures the intensity of illumination of the surface.

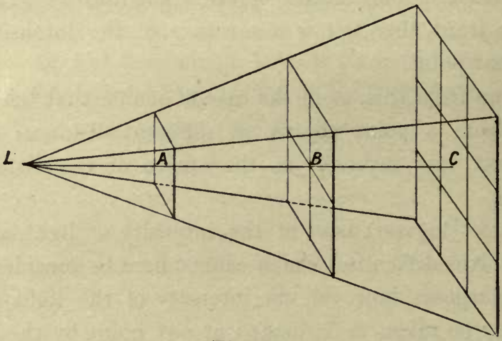

Fig. 18.

When a surface is illuminated by a luminous point as source of light, the intensity of illumination obviously varies inversely as the square of the distance of the surface from the source. Thus, let L (Fig. 18) represent the source of light, and consider the light diverging from L within the square pyramid indicated in the figure. If this pyramidal beam of light falls directly upon a screen, the illuminated surface will be a right section of the beam—that is, a square, as shown at A, B, and C in the figure. Now it can be seen, by comparing similar triangles, that the length of the side of any right section of the beam varies directly as its distance from L, and that the area of the section, therefore, varies directly as the square of its distance from L. Also, wherever the screen is placed the same quantity of light falls upon the area illuminated by this beam of light, but the area itself varies *directly* as the square of the distance from the source. The quantity of light which falls upon *unit area* of the illuminated surface must, therefore, vary *inversely* as the square of the distance of the surface from the source. Thus, if the screen be placed as at A, B, and C in the figure, at distances of 1, 2, and 3 units from L, the areas illuminated by the beam of light are obviously, as shown in the figure, 1, 4, and 9 units of area respectively, and as the quantity of light per second which falls upon A is spread over four times the area at B and over nine times the area at C, the quantity of light falling *on unit area*—that is, the intensity of illumination of the surface is, therefore, at B and C respectively, one-fourth and one-ninth of its value at A.

Experiment 9a.—Take a small square frame of wire and hold it half way between a small source of light and a screen, with its plane parallel to the square. A square shadow of the frame will be thrown on the screen. Measure the side of this square. It will be found that each side of the shadow square is twice the length of the corresponding side of the frame, and the area of the shadow square is, therefore, four times the area of the frame. Thus, if the shadow square be divided by median lines into four equal squares, each one of them is a square equal in area to the wire frame.

Similarly, if the screen be placed so that its distance from the source is three, four, five, &c., times the distance of the frame from the source, the area of the shadow square will be nine, sixteen, twenty-five, &c., times the area of the wire frame.

That is, if the position of the wire frame be supposed fixed, and the screen is placed at different distances beyond it, the area of the screen illuminated by the light passing through the frame varies directly as the square of the distance of the screen from the source of light, and the intensity of illumination, therefore, varies inversely as the square of the distance.

The intensity of illumination of a surface varies also with the angle of incidence of the light on it. Thus, if a conical beam of light from a point source is incident *directly* on a surface, a circular area, equal to a right section of the beam, is illuminated, but if the beam is incident *obliquely* on the surface, an elliptical area, equal to the oblique section of the beam, is illuminated. If the screen is at the same distance from the source in the two cases, so that the two sections of the cone cut the axis at the same point, the area of the oblique elliptical section is greater than that of the right circular section. It follows from this that when the light is obliquely incident on the surface it illuminates a surface of greater area than when directly incident, and the intensity of illumination of the surface is, therefore, greater for direct than for oblique incidence. It can be shown that the intensity of illumination is directly proportional to the cosine of the angle of incidence of the light.

16. **Illuminating Power of a Source of Light.**—The illuminating power of a source of light is directly proportional to the intensity of illumination which it produces on a white screen placed so that the light from the source is directly incident on it. The intensity of illumination produced by a source of light on a screen depends, however, on the distance of the screen from the source, so that it is usual to define the illuminating power of a source of light as measured by the intensity of illumination produced on a screen at **unit distance** from the source.

Hence, if I denote the illuminating power of a source of light —that is, the intensity of illumination which the source gives at unit distance from a screen—then the intensity of illumination which the source gives at a distance of D units from the screen is, by the law of inverse squares, $\frac{I}{D^2}$.

17. **Photometry.**—Photometry is the experimental quantitative comparison of the illuminating powers of different sources

of light; and the different forms of apparatus by which this comparison is effected are called **photometers.** The practical standard of comparison employed in photometry is the illuminating power of a standard sperm candle, six to the pound, burning 120 grains per hour. The illuminating power of any source of light is, therefore, generally expressed as being equivalent to that of a certain number of standard candles.

A more accurate standard of illuminating power is that given by the **pentane lamp** known as the Vernon-Harcourt pentane standard, in which a definite mixture of pentane vapour and air is burnt at a special burner. The illuminating power of the flame is about 10 candle power.

As already stated, the eye, although it is unable to compare different intensities of illumination quantitatively, is able to decide accurately whether two adjacent surfaces are equally or unequally illuminated. For this reason nearly all methods of photometry depend upon the adjustment of the illumination of two adjacent white screens to equality, and the details of the construction of photometers are devised to facilitate this adjustment.

If two adjacent portions of a white screen are equally illuminated by different sources of light at distances D_1 and D_2 from the screen, then, if I_1 and I_2 denote, respectively, the illuminating powers of these sources, the intensity of illumination of the equally illuminated portions of the screen is $\frac{I_1}{(D_1)^2}$ for one portion and $\frac{I_2}{(D_2)^2}$ for the other portion, and, therefore,

$$\frac{I_1}{(D_1)^2} = \frac{I_2}{(D_2)^2}, \text{ or } \frac{I_1}{I_2} = \left(\frac{D_1}{D_2}\right)^2.$$

That is, the illuminating powers of the sources of light are directly proportional to the squares of their distances from the screen, when these distances are so adjusted that each source gives separately the same intensity of illumination on the screen.

In all cases the light from each source is supposed to be directly incident on the portion of the screen which it illuminates, or, if the incidence is oblique, the angle of incidence is supposed to be the same for the light from each source.

All photometric measurements should be made in a dark room with dead black walls.

18. **Rumford's Photometer.**—In Rumford's photometer the sources of light to be compared are arranged so as to throw the shadow of a rod on a white screen. This arrangement obviously gives two areas on the screen, each of which is illuminated by only one of the sources; the shadow cast by either of the sources is illuminated by the other, while the screen, as a whole, is illuminated by both sources.

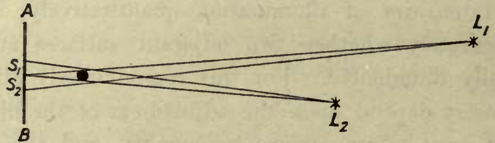

Fig. 19.

A plan of the arrangement usually adopted for this photometer is shown in Fig. 19. A vertical rod is set up a short distance in front of the screen AB, and the sources to be compared are placed as at L_1 and L_2, so that the shadows, S_1 and S_2, which they throw upon the screen are very close together, or, if possible, in actual contact along a common edge. The depth of the shadows—that is, the intensity of illumination of the shadow areas—can thus be easily compared, and the distances of the sources L_1 and L_2 can be adjusted until the shadows are of equal intensity. In order that the boundary line between the two contiguous shadow strips may be sharply defined, the shadows must have practically no penumbra—that is, the width of the sources of light parallel to the screen must be very small, and the rod must be fairly close to the screen. In making adjustments the edges of the shadows are most conveniently brought into *exact* coincidence by a small displacement of the screen relative to the rod as a fine adjustment. If L_1 and L_2 represent the final positions of the sources of light when the adjustment for equal intensity of illumination in the shadow areas is complete, and I_1 and I_2 denote respectively the illuminating powers of these sources, we have—

$$\frac{I_1}{I_2} = \left(\frac{L_1S_1}{L_2S_2}\right)^2.$$

The adjustment should be made under conditions such that L_1S_1 and L_2S_2 are equally inclined to the screen. Under these conditions, if D_1 and D_2 denote respectively the *perpendicular distances* of the sources at L_1 and L_2 from the screen, we get

$$\frac{I_1}{I_2} = \left(\frac{D_1}{D_2}\right)^2$$

for it can be seen by similar triangles that

$$\frac{L_1S_1}{L_2S_2} = \frac{D_1}{D_2}.$$

19. **Foucault's Photometer.**—This photometer, sometimes known as **Bouguer's photometer**, is exactly similar in principle to Rumford's photometer. The rod of Rumford's photometer is replaced by a partition P, arranged, as shown in Fig. 20, at right angles to the screen, and set up so that it can be moved in the direction of its length either by hand or by means of an adjusting screw. The sources of light are placed, one on each side of the plane of the partition, so that one-half of the screen is illuminated by one source, and the other half illuminated by

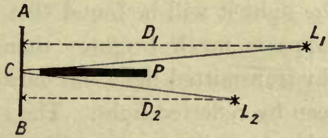

Fig. 20.

the other source. In the figure the half AC is illuminated by the source at L_1, and the half BC by the source at L_2; or, we may say, the half BC is in shadow from the source at L_1, and the half AC is in shadow from the source at L_2.

The distances of the sources from the screen are adjusted until the two halves of the screen are equally illuminated, and when this adjustment is made we have, as before,

$$\frac{I_1}{I_2} = \left(\frac{D_1}{D_2}\right)^2.$$

Where D_1 and D_2 denote the perpendicular distances of L_1 and L_2. respectively, from the screen. The common boundary at C of the two separately-illuminated halves of the screen can be adjusted to exact coincidence by moving the partition nearer to, or further from, the screen.

20. **Bunsen's Grease-spot Photometer.**—This photometer, devised by Bunsen, makes use of the fact that a spot of oil or grease on a sheet of paper is more transparent than the rest of the paper. If a small drop of colourless oil or grease be placed on a sheet of white unglazed paper, it is immediately absorbed, and a **grease-spot** forms on the paper. If the sheet be then held up to the light it will be found that the grease-spot is translucent and appears much brighter than the rest of the paper when seen by transmitted light, but darker than the rest of the paper when seen by reflected light. The explanation of this is simple. The colourless oil or grease fills up the interstices in the thickness of the paper and thus renders the region of the grease-spot more transparent than the rest of the paper. Hence, when the sheet is held between the source of light and the eye, light passes freely through the grease spot to the eye, and it is seen as a bright spot on the sheet. On the other hand, when the spot is viewed from the same side as the source, and so seen by reflected light, it is seen as a dark spot, because a large proportion of the light incident upon it passes through and

is not spent in illuminating the surface as at other points on the sheet.

It will be clear from what has been said, that if a thin paper screen with a grease-spot at its centre be placed between two sources of light, A and B, at right angles to the line joining them, its position can be adjusted so that the spot and the rest of the screen appear, when seen from either side, to be equally illuminated.

When this adjustment is made, the screen must be equally illuminated on both sides. Thus, if the screen be looked at from the same side as the source A, the light which comes through the spot from the source B exactly compensates, when both sides of the screen are equally illuminated, for the light from the source A which passes through the spot instead of being dispersed at its surface. Similarly, if the screen be looked at from the same side as the source B, the light which comes through the spot from the source A compensates exactly for the light from the source B, which passes through the spot without being diffused from its surface.

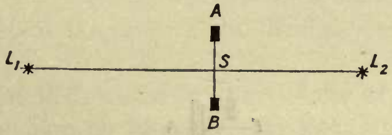

Fig. 21.

The light that reaches the eye from the surface of the spot is thus, when both sides of the screen are equally illuminated, the same as that which reaches the eye from any other equal area on the screen; and the spot is therefore, under these conditions, indistinguishable in point of illumination from the rest of the screen.

Hence, if L_1 and L_2 (Fig. 21) represent the positions of two sources of light to be compared, and AB the position of the screen when finally adjusted, so that the grease-spot S and the rest of the screen appear, when viewed from either side, to be equally illuminated, we have—

$$\frac{I_1}{I_2} = \left(\frac{L_1S}{L_2S}\right)^2.$$

Where I_1 and I_2 denote, as before, the illuminating powers of the two sources.

21. **Joly's Photometer.**—This photometer consists of two equal rectangular blocks of paraffin wax fitted closely together, but separated by a thin smooth sheet of tin foil, as shown at F in the horizontal section given in Fig. 22, and mounted in a small wooden case, AB, open at the ends, as shown in the vertical section through EA given in the figure.

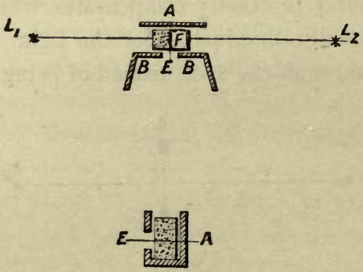

Fig. 22.

This double block of wax, with the central partition of tin foil, is placed between the sources of light to be compared, with the foil partition at right angles to the line joining them, and its position in this line is adjusted until the internal illumination of the wax as seen through the sides of the two blocks, by means of the small window shown at E, appears to be the same on each side of the foil. The light coming from each source enters the wax and is scattered in the wax, both before and after reflection, from the surface of the tin foil. The foil is opaque, so that one block is illuminated solely by one source of light, and the other block solely by the other source. Thus, if L_1 and L_2, in Fig. 22, represent the final positions of the two sources of light, the left-hand block is illuminated solely by the source L_1, and the right-hand block by the source L_2, and as the two blocks are equally illuminated, we have

$$\frac{I_1}{I_2} = \frac{(L_1F)^2}{(L_2F)^2}.$$

22. **Flicker Photometers.**—In photometers of the Flicker type a white surface is illuminated alternately, in rapid sequence, by the sources of light to be compared. When the lights are so placed that the intensities of illumination they produce on this surface are unequal, a flickering effect is visible to an eye looking at the surface. Hence, in comparing two sources of light, the relative distances of the sources from the surface are adjusted until this flickering vanishes, and the usual law is then applied, as above, to obtain the ratio of the illuminating powers of the sources.

23. **Effect of Difference in the Colour of the Sources under Comparison by Photometric Methods.**—When two sources of light, whose illuminating powers have to be compared, are appreciably different in colour, the photometric methods described above do not give satisfactory results. It is found impossible to judge, with any accuracy, of equality of illumination when the illuminated surfaces under comparison differ in colour.

Experiment 10.—Set up a Rumford's photometer and try to compare the illuminating powers of an incandescent gas-light and an ordinary candle.

It will be found that the shadow illuminated by the gas light is of a greenish-white colour, while that lit by the candle is of a reddish-yellow colour.

It will be found very difficult to decide when the two shadows are of equal intensity, and if several comparisons are made, as accurately as the observer can judge, it will be found that the values obtained for the ratio of the illuminating powers differ widely.

Experiment 11.—Try and compare the illuminating power of the moon and a street gas lamp, by comparing the shadows of a lead pencil thrown by these sources on a sheet of white paper.

It will be found that the shadow illuminated by the moon appears distinctly blue when compared with the reddish-yellow shadow illuminated by the lamp, and that, as in the last experiment, it is practically impossible to make anything like an accurate comparison of the illuminating powers of the sources.

The comparison of the illuminating powers of sources of light differing in colour can be made satisfactory only by a complicated colour method which cannot here be considered. The corresponding colour constituents of the light from the two sources are compared separately, and a comparison of the illuminating powers of the sources is deduced from the results so obtained.

It is found, however, that sources of light differing in colour can be readily compared by means of a **Flicker photometer,** and that the results obtained agree fairly well with those given by more elaborate methods.

24. **The Law of Inverse Squares.**—The law of inverse squares, deduced from theoretical considerations in Art. 15, is the basis of calculation in the photometric methods described above. If, however, we take two sources of light, for which the ratio of their illuminating powers is known, and determine their ratio experimentally by one of the photometric methods referred to, the agreement of the value of the ratio determined by experiments with the known value gives an *experimental* verification of the law of inverse squares.

Experiment 12.—Set up a Rumford, Bunsen, or Joly photometer, and compare the illuminating power of the light given by one candle with that of the light given by four exactly similar candles grouped together as one source.

The ratio of the illuminating powers should evidently be about 1 : 4, and it will be found, when the experimental comparison is made, that the ratio of the distances of the two sources from the screen will be 1 : 2, and the ratio of their illuminating powers is, therefore, given by the law of inverse squares as $1 : 2^2$ or 1 : 4.

Similarly, if the light from a three-candle source be compared with that from a five-candle source, the ratio of the illuminating powers of the sources will be given by experimental comparison as 3 : 5—that is, the distances of the sources from the screen when finally adjusted in position would always be found to be in the ratio $\sqrt{3} : \sqrt{5}$.

Note.—It is important in these experiments to arrange that the distances of the sources from the photometer are great compared with the dimensions of the sources themselves.

The consistency of the results obtained by experimental photometric methods is in itself a proof of the truth of the law of inverse squares which is involved in the theory of the methods.

25. **The Illuminating Power of the Sun.**—The illuminating power of the sun or any other celestial source of light is usually measured, not by the intensity of illumination that would be given by the source on a screen at unit distance from it, but by the intensity of illumination produced at any point on the earth by the source at its actual distance from the earth. Thus, the illuminating power of the sun at its distance of nearly 93,000,000 miles from the earth is found to be equal to about 600,000 foot-candles—that is, the intensity of the illumination given by the sun at the earth's surface is equivalent to about 600,000 times the illumination given by one candle at a distance of one foot from the illuminated surface.

CHAPTER IV.

REFLECTION AT PLANE SURFACES.

26. **Reflection of Light.**—When Light travelling in any medium is incident on the surface of another medium, the propagation of the wave motion is interrupted at the surface of separation of the two media, and three effects are, in general, observed to take place. One portion of the incident light is turned back or **reflected** at the surface, a second portion is **absorbed** at the surface, and a third portion travels on, or is **refracted** into the second medium. The reflection at the surface is, in general, associated with **scattering** or **diffusion** of the incident light, so that light is reflected or diffused *in all directions* from every point on the surface. This is generally known as **irregular** or **diffuse reflection.** If, however, the surface is so smooth that the inequalities on it are small compared with the wave length of light, the diffuse reflection becomes regularised so that the disturbance incident in any direction on the surface is reflected at the point of incidence in one definite direction related to the direction of incidence by a general law. This is usually called **regular reflection.** The amount of absorption which takes place at any surface depends upon the molecular character of the surface. For appreciable absorption to take place, the surface must be rough, for a smooth surface would reflect regularly, practically all the light incident on it; but whether a rough surface absorbs or not, or whether it absorbs much or little, depends entirely on its molecular structure. Thus the surface of white paper diffuses practically all the light that falls on it and absorbs little or none, the surface of a dead

black paper, on the other hand, absorbs all the light incident on it, and the surface of paper which appears coloured in daylight absorbs the constituents of white light unequally, absorbing some more or less completely, and diffusing what is not absorbed. The amount of light which enters the second medium is the residue left after reflection and absorption at the surface. The depth to which this residue penetrates depends (Art. 6) upon the degree of transparency of the medium; it may be absorbed in travelling through a very small thickness, or it may travel on through the medium without appreciable loss by absorption.

In diffuse reflection the wave disturbance which travels along any straight line to the surface is not reflected, merely with change of direction, along another straight line, but sets up at the point of incidence a fresh disturbance which travels out in all directions from the point. In regular reflection, however, the disturbance which travels along any straight line to the surface merely undergoes abrupt change of direction at the point of incidence, and travels on in a new direction which is definitely related, by the laws of regular reflection, to the direction of incidence.

Hence, if a ray of light suffers diffuse reflection at a surface, it may be supposed to give rise at the point of incidence to rays diverging *in all directions* from this point. If, however, a ray of light suffers regular reflection at a surface, it merely undergoes change of direction at the point of incidence, and travels on undisturbed in a new direction definitely related to the direction of the incident ray.

It will be seen that diffuse reflection is the cause of the visibility, in the presence of a self-luminous source, of objects which are not in themselves luminous. The light from the source which falls upon any object suffers diffuse reflection at the surface of the object, so that from every point on the surface, light is diffused in all directions just as if the surface were itself a source of light. In this way light enters the eye, wherever it may be placed, from every point on the object, and so renders the object visible from all points.

When, however, a regularly reflected pencil of light enters the eye, the surface at which reflection takes place is not seen at all, but the luminous point from which the pencil originates is seen at the geometrical focus of that portion of the pencil which enters the eye.

Ordinary daylight, as distinct from direct sunlight, is due to diffuse reflection of the sun's rays at the surfaces of the clouds and other objects on the surface of the earth; in the same way twilight is due to the diffuse reflection of sunlight from clouds and floating particles in the upper air, which are illuminated directly by the sun for some time after it has set below the horizon.

Experiment 13.—Admit a beam of sunlight through a small hole into a dark room, and let it fall on a sheet of white drawing paper.

The beam undergoes diffuse reflection at the surface of the paper, and it will be seen that although the incident beam can be clearly traced by means of the brightly illuminated dust particles which mark its path, no definite reflected beam can be traced. The reflected light is given off in all directions from the surface, which can be seen from any point in front of the plane in which it lies, as a brightly illuminated white surface. It will be noticed, too, that this surface lights up the room slightly, and acts in this respect in exactly the same way as a faintly self-luminous surface.

Now remove the paper, and let the beam fall on the surface of water in a glass vessel.

The beam now undergoes regular reflection at the surface of the water, and it will be found that the reflected beam consists of a single reflected beam, similar to the incident beam, and that the direction of this beam, as indicated by the bright dust particles which mark its path, is in the same plane as the incident ray and the perpendicular to the surface of the water at the point of incidence, and makes the same angle with this perpendicular as the incident ray.

Further, if a few drops of milk, or, better, a few drops first of silver nitrate and then of hydrochloric acid, be mixed with the water, so as to make it slightly turbid, the beam of light which enters the water and travels on into it can be clearly traced by the aid of the brightly illuminated white particles which mark its path. It will also be noticed, as explained in Art. 10, that the path of this beam is not a straight line continuation of the path of the incident beam, but that there is a sudden change of direction at the surface of the water.

If, now, the water is removed and the beam allowed to fall on the surface of clean, bright mercury in a small vessel, it will be found that the incident beam is regularly reflected at the surface, and that, as the incident light is practically all reflected, the reflected beam is nearly as strong as the incident beam, and much stronger than when reflection takes place at the surface of water where only a small part of the incident beam is reflected. When only a part of the incident beam is regularly reflected at any surface the reflection is said to be partial reflection.

The proportion of the incident light regularly reflected at any surface depends upon the nature of the media separated by the surface, the degree of smoothness of the surface, and also upon the angle of incidence of the light.

Experiment shows that polished silver reflects about 90 per cent., and a clean mercury surface about 67 per cent., of light *directly* incident on these surfaces. Transparent substances reflect a much smaller proportion of the incident light; a polished glass surface in air reflects only 4 per cent., and the surface of water only 2 per cent., of directly incident light. The reflecting power of a surface is found to increase with the angle of incidence, particularly in the case of transparent substances; thus, when the angle of incidence is $89\frac{1}{2}^\circ$ (nearly *grazing* incidence) the surface of clean mercury and the surface of water both reflect about 72 per cent. of the incident light. Even with comparatively rough surfaces, such as the surface of paper, polished wood, ordinary metal surfaces not specially polished, which practically do not reflect at all at ordinary angles of incidence, a very good image can be seen by reflection if the angle of incidence is nearly 90°.

27. **Mirrors.**—Any polished surface of regular geometrical form, which is smooth enough to reflect light regularly, is a **mirror**. The mirrors most commonly used are **plane** or **spherical** in form, but **cylindrical** and **parabolic** mirrors are employed for special purposes. The highest degree of smoothness or polish is necessary for an optical mirror; the inequalities of the surface must be small compared with the wave length of light, and the longest wave length is not more than ·00008 cm.

The first mirrors used were polished metal surfaces, and mirrors of this type are still used for certain optical purposes.

Mirrors of accurate form, intended for scientific purposes, are now usually made of glass coated in front with silver; the surface of the glass is worked to the required form and then covered with a thin layer of silver which is polished to the highest possible degree of smoothness. Mirrors of this type are generally called **specula**.

The plane mirrors in common use as looking glasses are made of plate glass backed with a thin coating of deposited silver. The silver is deposited, by a chemical reaction, on the polished surface of the glass, and the inner face of the coating, seen through the glass, gives an excellent reflecting surface.

28. **Definitions.**—The **normal** to a surface at any point is a line at right angles to the plane tangential to the surface at that point. Hence, if the surface is plane, the normal at any point is at right angles to the surface; and if the surface is spherical, the normal at any point is given by the radius drawn through that point.

When a ray of light meets a reflecting surface, the ray is said to be **incident** on the surface, and the point at which the ray meets the surface is **the point of incidence** of the ray. The angle which the **incident ray** makes with the normal at the point of incidence is called **the angle of incidence**, and the angle which the reflected ray makes with the normal at the point of incidence (and reflection) is **the angle of reflection**.

The plane determined by the incident ray and the normal at the point of incidence is **the plane of incidence**, and, similarly, the plane determined by the reflected ray and the normal is **the plane of reflection.**

29. **Laws of Reflection.**—The laws of regular reflection of light can be deduced from the wave theory of light. They can also be established experimentally by very careful and accurate experiments of the type illustrated by the following simple experiments :—

Experiment 14.—The apparatus required for this experiment is shown in Fig. 23. A graduated circle fixed in a vertical plane has a small plane mirror, M, attached to it in a horizontal position so that its reflecting surface contains the centre of the divided circle.

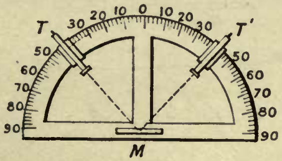

Fig. 23.

It carries also two sighting tubes T and T′, fitted so that they can be moved round the circumference of the divided circle with their axes parallel to the plane of the circle, and directed in all positions towards the centre of the circle. The positions of the axes of the sighting tubes relative to the graduations of the divided circle are indicated by index marks attached to the slides carrying the tubes. The circle is graduated, as shown in the figure, from a zero opposite the normal to the mirror at the point where the axes of the tubes intersect.

Take this apparatus and fix one tube in any position, as at T in the figure. Now move the other tube T′ round the circle and look through it as it is moved.

It will be found that in a certain position it is possible to see through the tube T, and to recognise objects some distance from the apparatus on the axis of this tube. That is, light from these objects is incident on the mirror through the tube T, and is reflected at the mirror along the tube T′.

This shows that the direction of the reflected light is in the same plane as the direction of the incident light and the normal to the mirror at the point of incidence, for the axes of the two tubes and the normal to the mirror are all in the same plane. That is, the plane of reflection is coincident with the plane of incidence.

Now select a bright point on one of these objects, and fix the tube T′ in a position such that it is seen exactly at the centre of the circular field of view through the tubes. Read the angles which the axes of the tubes make with the normal to the mirror. It will be found that these angles are always equal whatever their actual values may be.

Hence, we infer that the angle of reflection is equal to the angle of incidence and that both angles are in the same plane.

Experiment 15.—Take a rectangular strip of plate glass* and black one face by painting it with a mixture of lampblack and gum. The other face of the strip can now be used as a reflecting surface, without the confusion that would be caused by reflection from the face which has been blackened. Light incident through the glass on this face is practically all absorbed by the coating of lampblack.

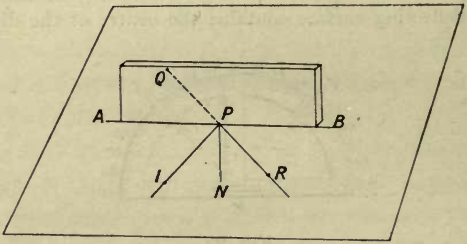

Fig. 24.

Set up the strip so that it stands vertically on one edge on a sheet of paper, as shown at AB on Fig. 24. Draw with a pencil the line AB, marking on the paper the trace of the reflecting face of the strip. Make a dot on the paper at any point I, and another dot at any point P, on the line AB.

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* A strip of unblackened glass is troublesome. Reflection takes place at both surfaces of the glass, and this leads to confusion in the experiment. If the glass is thin the two reflecting surfaces are close together and the confusion is minimised, but it is difficult to get the strip to stand on its edge without mounting it on a piece of cork, and the surface of thin glass is often rather uneven. A strip of good mirror glass off which the backing has been dissolved by methylated spirit, and the exposed silvered surface carefully polished with rouge, gives a satisfactory reflecting surface.

Light incident on the reflecting surface from the point I in the direction IP will be reflected at this point, say, along PR, and if the eye be placed at R so as to receive this reflected light, the dot at I will be seen (Art. 10) at a point Q on RP produced behind the reflecting surface.

Hence, let the eye be placed just at or a little above the level of the paper, and adjust its position so that the point P and the image at Q of the dot at I are in the line of sight; then make a dot, R, on the paper, so that the points R and P are exactly in a line with the image seen at Q.

Remove the strip of glass and draw the lines IP and PR, and the line PN perpendicular to AB at the point P.

The line PN is evidently the normal to the reflecting surface, represented by AB, IP represents an incident ray of light, and PR the corresponding reflected ray. The angle IPN is, therefore, the angle of incidence and NPR the angle of reflection.

Measure these angles carefully for a number of different positions of the point I. It will be found in all cases that the angle IPN is equal to the angle NPR. The lines IP, PN, and PR are also all in one plane, the plane of the paper.

Hence the law of reflection indicated by this experiment is that the incident and reflected rays, and the normal to the reflecting surface at the point of incidence, are in the same plane, and the angle of reflection is equal to the angle of incidence.

Pins stuck vertically into the drawing board may be used instead of dots, but they are much more troublesome to use.

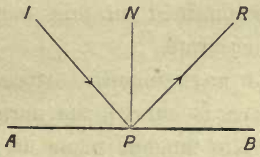

Fig. 25.

Hence, when a ray of light is incident on a reflecting surface, as shown in Fig. 25, it is reflected in accordance with two laws which may be thus formulated :—

1. The angle of reflection, IPN, is equal to the angle of incidence, NPR.

2. The angles of incidence and reflection are in the same plane.

These may be expressed as one law, as follows:—The angles of incidence and reflection are equal, and are in the same plane.

The significance of the statement that the angles of incidence and reflection are in the same plane will be realised more fully by the aid of the following experiment:—

> **Experiment 16.**—Take a large, flat cork and stick a pin in it, in a slanting direction, at any point on its flat surface, to represent a ray of light incident on the surface. At the same point stick in another pin, perpendicular to the surface, to represent the normal to the surface at the point of incidence. Now stick in another pin at the same point to represent the reflected ray. This pin must, according to the law given above, be in the same plane as the two other pins, and must be inclined to the pin representing the normal at the same angle as the pin representing the incident ray. It will be realised in placing this third pin that there are many positions in which it makes the proper angle with the normal, but that only one of these positions is in the same plane as the two other pins. Mark the plane of incidence and reflection by means of a sheet of paper, and draw a line on the surface of the cork marking the trace of this plane.

The strongest proof of the laws of reflection lies in the fact that in numerous experiments these laws are assumed to be true, and the results obtained on this assumption are always found to be strictly accurate.

The laws hold for any smooth surface whether plane or curved. If the surface is curved the normal at any point is the perpendicular to the tangent plane at that point.

30. **Images.**—When a luminous body is viewed directly, pencils of light from every point on the body enter the eye, and thus the body is seen, and its form defined as the locus of the foci of the pencils of light entering the eye. If, however, from any cause these pencils suffer change of direction, so that

on entering the eye they actually come from, or appear to come from, foci other than the points on the luminous surface from which they start, the locus of these foci is called the **image** of the luminous body.

An image may be either **real** or **virtual.** In the case of a real image, the rays entering the eye actually do come from the points which make up the image—that is, the foci of the pencils of light which enter the eye are real, and the image is the locus of these real foci. In the case of a virtual image, however, the rays of light entering the eye only appear to the eye to come from the points which make up the image—that is, the foci of the pencils of light which enter the eye are virtual, and the image is the locus of these virtual foci.

A real image differs from a luminous body in the fact that pencils of light diverge from the latter in all directions, whereas they diverge from the former only in the directions taken by the rays involved in its formation.

31. **Reflection of Light from a Luminous Point at the Surface of a Plane Mirror.**—Let L in Fig. 26 represent the luminous point, and MM the trace of the reflecting surface of a plane mirror at right angles to the plane of the paper. Consider any ray, LA, coming from the point L, and incident on the mirror at A. The plane of incidence is here the plane of the paper for the normal AN at the point A, and the incident ray LA are both in this plane. The reflected ray AR will, therefore, lie in the plane of the paper, and its direction will be such that the angle of reflection RAN is equal to the angle of incidence LAN. Similarly, any other ray, LA′, will be reflected along AR′, so that the angle R′A′N′ is equal to the angle LA′N′.

Now, if an eye be placed at RR′ to receive the pencil of rays diverging from L and bounded, after reflection at AA′, by the rays LAR and LA′R′, the point L will be seen apparently at L′, the geometrical focus of that portion, AA′RR′, of the pencil which enters the eye. The point L′ evidently lies at the intersection of RA and R′A′, and it can be shown that it is on the normal to the mirror surface through L, and at the same distance behind the surface as the point L is in front of it. Thus, through L draw LOP normal to the mirror at O, and produce RA to cut this normal at L′. Then, since the angles ALO and AL′O are equal respectively, by *Euc.* i., 29, to the angles LAN and RAN, it follows that they are equal to each other. The triangles AOL and AOL′ are, therefore, by *Euc.* i., 26, equal in every respect, and the side OL′ is equal to the side OL—that is, the point L′ is at the same distance behind the mirror as the point L is in front of it. Similarly, it can be shown that the direction of any other reflected ray, A′R′, if produced backwards, cuts the normal through L at the same point L′—that is, *all* rays incident on the mirror from L diverge, after reflection, as if they came from a point L′ on the normal through L at the same distance behind the mirror as the point L is in front of it. For example, all the rays in the pencil LAA′ are reflected within the pencil AA′RR′, and diverge as if they came from the point L′ on the normal through L.

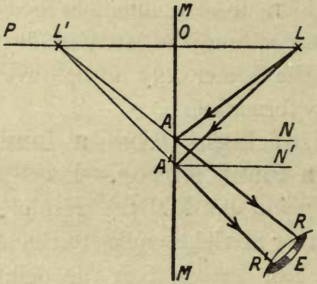

Fig. 26.

The point L′ is the **image** of the luminous point L, formed by reflection at the surface of the plane mirror MM. The image is a **virtual image**, for the reflected rays evidently do not *actually* diverge from L′, but any reflected pencil, such as AA′RR′, which really arises by reflection at the portion AA′ of the mirror, *appears* to the eye receiving it to come from the point L′ behind the mirror. It has thus been proved, by assuming the laws of reflection, that the image of a luminous point formed by reflection at a plane mirror is on the normal to the mirror through the point at the same distance behind the mirror as the point itself is in front of it. Hence, if this can be shown to be true experimentally, we get an indirect experimental proof of the laws of reflection.

Experiment 17.—Set up a strip of thin glass on a sheet of paper, as explained in Experiment 15. Stick a pin into the paper at A (Fig. 27) a few inches in front of the reflecting surface. An image of the pin, formed by the reflection at the surface of the glass strip, will be clearly seen * behind the strip at a point B. Place another pin at B, so that it coincides with this image. This can be done exactly by adjusting the position of the pin until it appears coincident with the image from all points of view; there should not, for example, be any relative displacement of the pin and image when the eye is moved from side to side. Remove mirror and pins, join AB, and show by actual measurement that the distances of the image and object from the reflecting surface are equal, and that the line joining them is perpendicular to the reflecting surface.

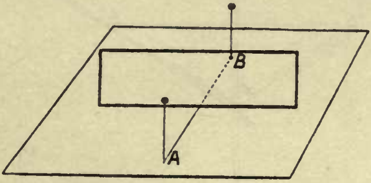

Fig. 27.

32. Reflection of a Convergent Pencil converging to a point behind a Plane Mirror.

—Let the convergent pencil BB′L, Fig. 28, converging to a point L behind the plane mirror MM, be incident on the mirror at AA′. It can be shown by the method of the foregoing article, that, after reflection at the surface of the mirror, the pencil will converge to a point L′ on the normal to the mirror through L, and at the same distance in front of the mirror as the point L is behind it. As before, the point L′ is the image of the point L, but, in this case, the image is a **real image**, for the reflected rays actually pass through the point L′, and would enter an eye placed as at E, as a pencil of rays actually diverging from this point. This image would appear as a bright spot of light on a screen placed at L′, or could be seen as a bright point at L′ by an eye placed, as at E, at a distance of 9 or 10 inches from L′. It must be remembered, however, that there is no real **source of light** at L′, only a real **image**.

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* The pin should be in bright light, but the background against which the image is viewed should be dark.

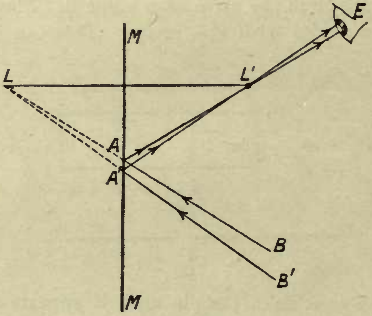

Fig. 28.

It has already been explained that a pencil of light from any luminous point is always a divergent pencil. It will be found later, however, that a divergent pencil may become convergent as the result of reflection or refraction under special conditions.

33. **Image of an Object formed by a Plane Mirror.**—The general geometrical construction for the determination of the image of an object by reflection at any reflecting surface, plane or curved, is based upon the following rule:—From any point on the object draw any two rays incident on the reflecting surface, and find, by application of the laws of reflection, the directions of these rays after reflection at the surface; the

geometrical focus of the two rays after reflection is then the image of the point on the object from which the two incident rays are drawn. This rule is the one applied in Art. 31, in determining the position of the point L′ as the image of the point L. The application of the rule is simplified by drawing the two incident rays in directions for which the directions of the reflected rays are easily found. In the case of spherical mirrors it will be found that it is always possible to select two incident rays, for which the directions after reflections are known, without any construction. In the case of a plane reflecting

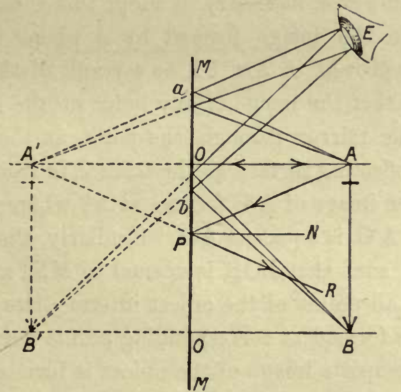

Fig. 29.

surface, only one such ray can be drawn; if the incident ray is normal to the surface, the direction of the reflected ray is evidently back along the normal. For any other incident ray the direction of the reflected ray can be found only by making the angle of reflection equal to the angle of incidence.

Thus, in Fig. 29, let AB represent a luminous object placed in front of a plane reflecting surface or mirror MM. From the point A, on the object AB, draw the two rays AO and AP incident on the mirror: the ray AO, incident along the normal to the mirror, is reflected back along the normal, and the ray AP, incident on the surface at an angle APN, is reflected along PR, making the angle of reflection RPN equal to the angle of incidence APN. The image of A is thus determined at A′, the virtual focus of the two reflected rays AO and PR. Similarly, the image of any other point, B, may be obtained, and the images of points AB, intermediate between A and B, may be assumed to lie on A′B′. Hence, A′A′ is the image of AB. When the form of the object is more complex than that considered here, the image of a number of points sufficient to determine the complex image must be obtained.

It is not, however, necessary to adopt this general construction to determine the image formed by a plane mirror. It has already been proved, in Art. 31, as a result of the application of this method, that the image of any point on the object is, on the normal to the mirror through the point, at the same distance behind the reflecting surface as the point is in front of it. Hence, in Fig. 29, the image of A is formed at A′, where AOA′ is normal to MM, and A′O is equal to AO. Similarly, the image of B is formed at B′, such that BOB′ is normal to MM, and B′O is equal to BO. For all points of the object intermediate between A and B, images are formed at corresponding points between A′ and B′, and thus a complete image of the object is formed at A′B′.

34. **Path of Rays by which an Image is seen.**—Any point on an image is seen by means of the pencil of light which really comes from the corresponding point on the object, and is incident on the mirror in such a direction that it is reflected from the mirror as if it came direct from the point seen on the image. Thus, in Fig. 26, the point L′, the image of L, is seen by means of the pencil of light from L, which is incident on the mirror at AA′, and is there reflected in a direction such that it enters the eye at E, as if it came direct from the point L′.

The construction for determining the path of this pencil is evident from the figure. Draw lines joining L′ to the points RR′, the extremities of a diameter of the pupil of the eye, and let these lines cut the mirror at A and A′; then join L to the points A and A′, and the paths LAR and LA′R′ are the boundaries of the bent pencil of light by which light from L enters the eye as if it came from L′. Similarly, in Fig. 29, an eye at E sees all points on the image A′B′, by pencils of light reflected from the portion *ab* of the mirror; the pencils by which the extreme points A′ and B′ are seen, are shown in the figure, and the lines A*a*E and B*bE* indicate the extreme rays of the beam of light from AB which reaches the eye at E, after reflection, at the portion *ab* of the mirror surface.

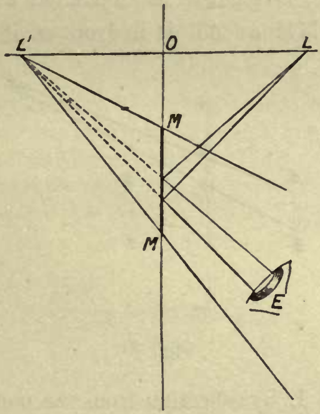

Fig. 30.

It will be obvious, from what has been said, that an image is formed by a plane mirror of a point placed anywhere *in front of the plane of the mirror*, and can be seen by an eye placed in any position, such that *the line joining it to the image cuts the mirror.* Thus, let MM (Fig. 30) represent a plane mirror, and L a luminous point placed anywhere in front of the plane in which the mirror lies. An image of L will obviously be formed at L′, so that LL′ is normal to the plane of the mirror, and LO equal OL′. This can be proved exactly, as in Art. 31; the figure, which corresponds to Fig. 26, shows the necessary construction, and shows also the path of the rays, by which an eye at E is able to see the image L′. The position of an eye to which the image L′ is visible must evidently be in front of MM, and between the lines drawn from L′ through the points MM.

If an object is placed anywhere in front of the plane of a mirror, it is possible to indicate the position and least extent of the mirror surface required in order that the image of the object may be seen by an eye placed at any given point in front of the mirror. Thus, in Fig. 31, if MM denote the trace of the plane of a mirror surface, and AB an object in front of it, then A′B′, the image of AB, is formed, as explained above, at the same distance behind MM as AB is in front of it, and this image is

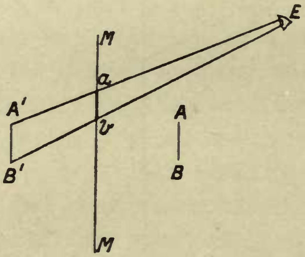

Fig. 31.

seen by an eye at E by reflection from the portion *ab* marked off on the mirror surface, by joining the point E to the extreme points A′ and B′ on the image A′B′. This portion *ab*, therefore, indicates the proportion and extent of the mirror surface necessary in order that the image A′B′ shall be seen by an eye at E.

Similarly, if the position and extent of the mirror surface are fixed, it is possible to indicate what portion of the image of any object in front of the mirror can be seen by an eye placed at any given point in front of the mirror. Thus, in Fig. 32, if A′B′ represent the image of an object, AB, in front of the mirror MM, then the portion of the image seen by an eye at E is that portion which lies between the points *a* and *b* determined by drawing the lines EM*a* and EM*b* from E, through the extreme points of the mirror, to cut the image A′B′ in *a* and *b*.

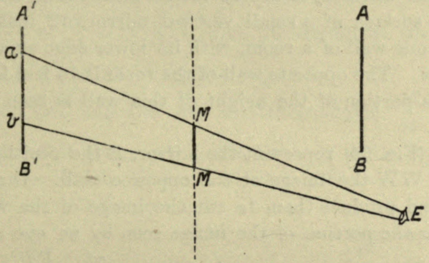

Fig. 32.

Numerical Examples.—1. A man stands in front of a vertical mirror and is just able to see a full length image of himself in the mirror. Find the position and length of the mirror.

In Fig. 33, let MM represent the plane in which the mirror is placed, AB the man, and A′B′ his image. Also let E represent the position

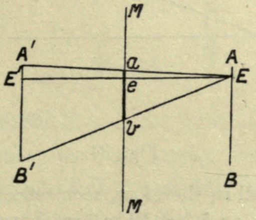

Fig. 33.

of the man's eyes, and join EA′ and EB′, cutting the plane of the mirror at *a* and *b*. Then *ab* indicates the length and position of the mirror surface necessary for the man to see his full length image, A′B′.

The triangles EA′B′ and E*ab* are similar, therefore we have $\frac{ab}{\mathrm{A'B'}} = \frac{\mathrm{E}a}{\mathrm{EA'}} = \frac{1}{2}$. That is, $ab = \frac{1}{2}\,\mathrm{A'B'} = \frac{1}{2}\,\mathrm{AB}$, or the necessary length of the mirror is one-half the man's height. Also, if E′ represent the image of E, the line E and E′ is normal to the mirror, and the mirror is so placed that its upper edge is a distance, *ea*, above this normal. Now, the triangle E*ea* and EE′A′ are similar, and, therefore,

$\frac{ea}{E'A'} = \frac{Ee}{EE'} = \frac{1}{2}$, or $ea = \frac{1}{2}\,E'A$. The upper edge of the mirror is, therefore, above the level of the man's eyes by a distance, *ea*, equal to half the man's full height above the level of his eyes.

2. A man whose eyes are 70 inches from the ground stands 4 feet from the surface of a small vertical mirror, 12 inches long, which hangs on one wall of a room, with its lower edge on a level with the man's eyes. The opposite wall of the room is 16 feet from the mirror, find which portion of the height of this wall is seen reflected in the mirror.

Let *ab* (Fig. 34) represent the mirror, E the position of the man's eyes, and WW the image of the opposite wall. Draw the lines E*a* and E*b* and produce them to cut the image of the wall at *c* and *d*. Then *cd* is the portion of the image seen by an eye at E. The triangles E*ab* and E*cd* are similar, therefore $\frac{cd}{ab} = \frac{Ed}{Eb} = \frac{20}{4} = \frac{5}{1}$. That is, $cd = 5\,(ab) = 60$ inches or 5 feet.

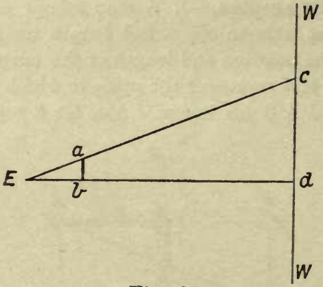

Fig. 34.

The opposite wall is, therefore, seen reflected in the mirror from a height of 70 inches, or 5 feet 10 inches above the floor, to a height of 10 feet 10 inches above the floor; the point to which *d* in the image corresponds is 70 inches above the floor, and the point to which *c* on the image corresponds is 5 feet higher.

3. In the last example, if the mirror is 9 inches wide and the man's eyes are 3 inches apart, find the width of the portion of the opposite wall which is seen reflected in the mirror, assuming the man to stand centrally opposite the width of the mirror.

Here if *ab*, Fig. 35,[*] represent the width of the mirror, *ee* the positions of the man's eyes, then *eac* is the extreme line of sight on one side, and *ebd* the extreme line on the other side; and *cd* is, therefore, the width of the strip of the image WW within the range of the man's vision.

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[*] The plane of this figure is a horizontal plane at the level of the man's eyes.

Draw, through O, the normal to the mirror cutting the image WW in *l*, *ab* in *m*, and *ee* in *n*. Then since O*ee* and O*ab* are similar triangles, $\frac{On}{Om} = \frac{ee}{ab} = \frac{3}{9} = \frac{1}{3}$, and the distances O*n* and O*m* are, therefore, 1 foot and 3 feet respectively.

Also since O*ab* and O*cd* are similar triangles, $\frac{cd}{ab} = \frac{Ol}{Om} = \frac{19}{3}$.
That is, since *ab* is given as 9 inches, CD = $\frac{19}{3}$ × 9 inches = 57 inches.

That is, the width of the wall seen reflected in the mirror is 57 inches or 4 feet 9 inches.

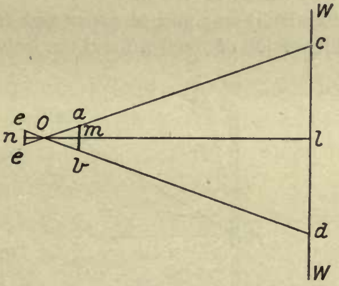

Fig. 35.

35. Lateral Inversion.—If we look at the reflection of the face in a looking-glass, we can easily verify that the image of the right eye forms the left eye of the reflected face, and the image of the left eye forms its right eye. Similarly, the image of the right hand and arm forms the left hand and arm of the reflection we see facing us in a mirror, and the image of the left hand and arm forms its right hand and arm. Hence, if a person, writing at a table, sees himself by reflection in a mirror he appears to be writing with his left hand instead of his right hand. These are particular instances of a general result of reflection known as **lateral inversion.** The image of every point on the object is formed at a point directly opposite to it in the mirror, and the front of the image always faces in a direction opposite to that of the object: the result of this is that points on the right and left of the object appear, respectively, on the left and right of the image.

Lateral inversion cannot affect the appearance of objects, such as the face and body, which are bilaterally symmetrical, but with non-symmetrical objects, such as printed or written characters, the effect is sufficiently evident and well known.

Experiment 18.—Print the letters MOT in block letters on a strip of paper and examine the image of the letters in a mirror. Note the lateral inversion of the *word:* the *letters* are symmetrical and do not suffer by lateral inversion.

Print the letters RED on a slip of paper and examine the image. Note the lateral inversion of the word and the letters.

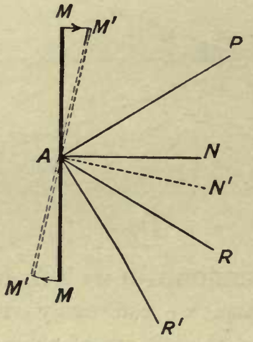

Fig. 36.

36. **Effect of the Rotation of a Mirror on the Direction of the Reflected Ray.**—Let MM, Fig. 36, denote the trace of a plane mirror, and suppose it to be rotated round an axis at A, perpendicular to the plane of the paper, through the angle MAM′ into the position M′M′. When the mirror is in the position MM a ray, PA, incident at A, is reflected, as shown in the figure, along AR, making the angle of reflection RAN equal to the angle of incidence PAN. When, however, the mirror is rotated through the angle MAM′, or the

equal angle NAN′, into the position M′M′, the same ray PA is reflected, as shown, along AR′, making the angle of reflection R′AN′ equal to the angle of incidence PAN′. The direction of the reflected ray has thus been changed through the angle RAR′, and it can be proved that this angle is twice the angle NAN′ through which the mirror has been rotated. Thus, we have—

$$\angle \mathrm{PAR} = 2 \angle \mathrm{PAN},$$

and $$\angle \mathrm{PAR}' = 2 \angle \mathrm{PAN}' = 2 \angle \mathrm{PAN} + 2 \angle \mathrm{NAN}'.$$

Hence, $$\angle \mathrm{PAR}' - \angle \mathrm{PAR} = 2 \angle \mathrm{NAN}';$$

that is, $$\angle \mathrm{RAR}' = 2 \angle \mathrm{NAN}'.$$

This shows that when the mirror is rotated through any angle the reflected ray is rotated through twice that angle.

37. **Reflection at Plane Surfaces inclined to each other.**—Before dealing with particular cases of reflection at two inclined plane surfaces, it will be helpful to consider the general principles applicable in all cases.

Imagine an object A, Fig. 37, placed between two mirrors M_1 and M_2 inclined to each other at any angle. An image of A will be formed by reflection at each mirror, giving the two images A_1' and A_2' in the mirrors M_1 and M_2 respectively. Hence light from A, after reflection at M_1, is incident upon M_2 as if it came from A_1', and is therefore reflected from M_2 as if from a point A_1'', which may be looked upon as the image of A_1'. Similarly, light from A, after reflection at M_2, is incident upon M_1 as if it came from A_2', and is therefore reflected from M_2 as if from a point A_2'', which may be considered as the image of A_2'. The points A_1'' and A_2'' are thus images of points A_1' and A_2', which are themselves images of A—that is, they are images formed by two successive reflections of the light from A; A_1'' is formed by reflection of the light from A first at M_1 and then at M_2, and A_2'' by reflection of the light from A first at M_2 and then at M_1. The images A_1' and A_2', formed by a single reflection, are called **first order** images, and

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A_1'' and A_2'',* formed by two reflections, are images of the **second order.**

In the same way, light from A, after reflection first at M_1 and then at M_2, is incident on M_1 as if it came from the point A_1'', and is therefore reflected from M_1 as if from a point A_1''', which may be considered as the image of A_1''. Also, it will be seen that light from A, after reflection first at M_2 and then at M_1, is incident upon M_2 as if it came from A_2'', and is therefore reflected from M_2 as if from A_2''', which may therefore be considered as the image of A_2''.

The points A_1''' and A_2''' are evidently images of the **third order,** for they are images of points which are themselves images of the second order, and are formed by three successive reflections of the light from A; A_1''' is formed by reflection of the light from A first at M_1, then at M_2, and then at M_1 again, and A_2''' is formed by reflection of the light from A first at M_2, then at M_1, and then at M_2 again.

This process of successive reflections at the surfaces of the two mirrors will evidently go on so long as the last formed image at either mirror is in front of the plane of the other. When, however, an image is formed for either mirror in the angle vertically opposite the angle between the mirrors, this image is behind the plane of both reflecting surfaces, and light reflected from either mirror as if from this image cannot be incident on the other mirror, and further reflection is therefore impossible. In the figure it will be seen the images A_1'''' and A_2'''' are formed in the angle vertically opposite the angle between the mirrors M_1 and M_2, and that the light reflected from M_2 or M_1 as if from those points cannot from its direction again be incident on either mirror.

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* The notation here adopted should be noted; the suffix figure indicates the mirror at which the *first* reflection takes place, and the number of dashes indicates the order of the image. Thus, A_2'''' denotes an image of the *fourth* order, formed by *four* reflections beginning at the mirror M_2.

It should be noticed that the images formed in this way can be arranged in two series, according to the mirror at which first reflection takes place. The series A_1', A_1'', A_1''', &c., is formed by one, two, three, &c., successive reflections, beginning at the mirror M_1; the series A_2', A_2'', A_2''', &c., is formed by one, two, three, &c., successive reflections, beginning at the mirror M_2; and the members of each series are so related that any one may be considered as the image of the one immediately preceding it in the series. It should be noted, too, that the *odd* members of each series are formed behind the mirror at which the *first* reflection takes place, while the *even* members are formed behind the other mirror.

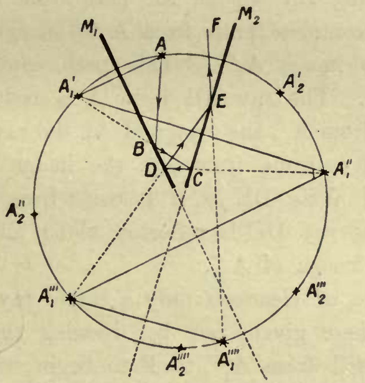

Fig. 37.

The path by which any particular image is seen—that is, the path by which light from A reaches the eye as if it came from that particular image—is found by tracing the path back through its successive reflections to A. Thus, in Fig. 37 the image A_1'''' is seen by an eye at F by light which travels from A along the path ABCDEF. The image A_1'''', which is of the fourth order, can be traced back through the images A_1''', A_1'', and A_1' to A; the image A_1'''' is the image of A_1''', A_1''' is the image of A_1'',

A_1'' is the image of A_1', and A_1' is the image of A. Hence, in order to draw the path by which the image at A_1'''' is seen by an eye at F, we first join F to A_1'''' and mark the portion EF, *in front* of the mirror, as the path by which a ray of light from A reaches the eye after its fourth reflection at E. Then, by joining the point E to A_1''' we get, in the same way, DE as the path of the ray from D to E after its third reflection at D. Similarly, by joining D to A_1'' we get CD as the path from C to D after the second reflection at C; and by joining C to A_1' we get BC as the path from B to C after the first reflection at B. Then, finally, by joining BA we get the path from A to B, and so determine the complete path from A to an eye at F which is looking at the image A_1''''. This path can now be easily traced from A. The ray AB from A is reflected along BC as if it came from A_1', the image of A; the ray BC is reflected along CD as if it came from A_1'', the image of A_1'; the ray CD is reflected along DE as if it came from A_1''', the image of A_1''; and the ray DE is reflected along EF as if it came from A_1'''', the image of A_1'''.

For the sake of clearness only a single ray path is shown in the figure here given, but by drawing the extreme rays of a small pencil from A_1'''' to F to begin with, the path of a small pencil of light from A to F can easily be found by the method described above.

It should be noted that the whole of the path from A to F lies in front of the mirrors; evidently no part of the actual path of a ray of light from A to F can lie behind the mirrors.

38. **Parallel Mirrors.**—Let A, in Fig. 38, represent a luminous point between the two parallel mirrors M_1 and M_2. Through A draw the normal to the two mirrors, cutting them, respectively, at N_1 and N_2. All the images of A which can be formed by successive reflections at the mirrors must evidently lie on this normal; and, in this case, there will be no limit, theoretically, to the number of images, for since the mirrors are

parallel, no image can have a position behind the planes of both mirrors.

The positions of the successive images on the normal are readily determined. The first reflection of a pencil of light from A, at the mirror M_1, gives rise to the first order image A_1' at a point on the normal such that $N_1A_1' = N_1A$; the second reflection of the pencil at M_2 gives the second order image A_1'' at a point on the normal such that $N_2A_1'' = N_2A_1'$; the third reflection of the pencil at M_1 gives the third order image A_1''' on the normal at a point such that $N_1A_1''' = N_1A_1''$, and so on indefinitely. Similarly, if we consider the successive reflections of a pencil of light from A, beginning with the first reflection at the mirror M_2 we should locate the series of images A_2', A_2'', A_2''' at points on the normal, such that the distances N_2A_2', N_1A_2'', and N_2A_2''' are equal respectively to the distances N_2A, N_1A_2', and N_2A_2''.

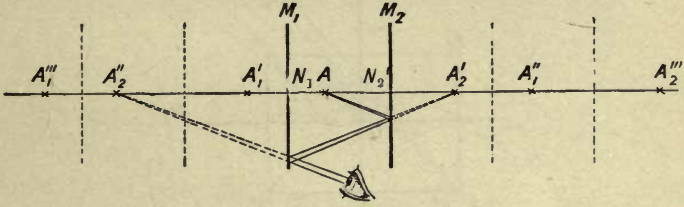

Fig. 38.

It will be seen from the figure that the two images of the *same order* are always on opposite sides of the mirrors, and that the distance between two images of the *n*th order is always 2*n* times the distance between the mirrors. It can also be verified that in each series the distance of an image, behind the mirror at which it is formed, increases by the distance between the mirrors at each reflection.

Although the number of images which may theoretically be formed by successive reflection between parallel mirrors is infinite, it must be remembered that an image of the *n*th order is seen by n reflections at the surfaces of the mirrors, and, as there is considerable loss of light, mainly by diffusion, at each reflection, the images get fainter and fainter as n increases, and ultimately become too faint to be visible.

The path of the pencil of light by which any image is seen, can be drawn by the method explained in the foregoing article. The path by which the image A_2'' is seen is shown in Fig. 38. The dotted lines which appear in the figure are useful in fixing the positions of the images.

39. **Mirrors Inclined at Right Angles to Each Other.** —Let A, in Fig. 39, represent a luminous point placed between

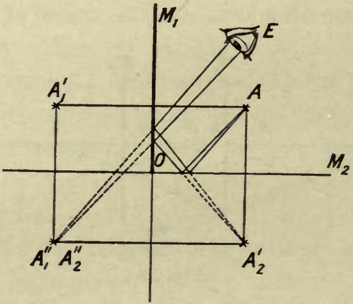

Fig. 39.

two plane mirrors OM_1 and OM_2, set at right angles to each other. Two first order images of A are formed at A_1' and A_2' by the mirrors OM_1 and OM_2 respectively. But A_1' lies in front of OM_2, and therefore a second order image is formed at A_1'' by that mirror; similarly A_2' lies in front of OM_1, and another second order image is therefore formed at A_2''. It is evident, however, from the geometry of the figure, that these two second order images must be coincident in position; the distances of the two points, A_1'' and A_2'', behind the plane of OM_1 and also behind the plane of OM_2 are equal, and correspond to the distances of the point A in front of these two planes, so that the two points must coincide, at the fourth corner of the rectangle of which the three other corners are at A, A_1', and A_2'.

An eye placed anywhere within the angle M_1OM_2 would see the three images, A_1', A_2', and A_1'' or A_2''. The images A_1'' and A_2'' are coincident, and cannot both be seen at the same time by a single eye; an eye within the angle M_1OA sees the image A_2'' in the mirror OM_1, while one within the angle M_2OA sees the image A_1'' in the mirror OM_2.

The path by which an eye at any point within the angle M_1OM_2 sees any one of these images of the point A, is readily found by the method already explained. The figure shows the path by which a pencil of light from A reaches an eye at E looking at the image A_2''.

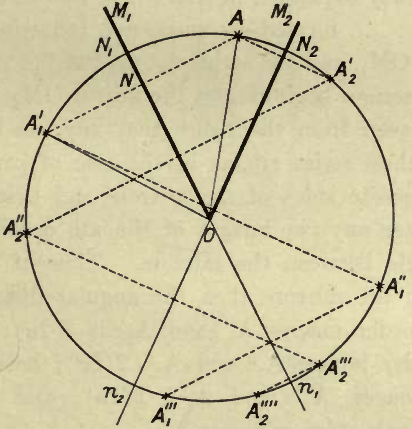

Fig. 40.

40. **Mirrors Inclined at any Angle.**—Let A represent a luminous point between two plane mirrors OM_1 and OM_2, inclined to each other at the angle M_1OM_2, as shown in Fig. 40. With O as centre and OA as radius, describe a circle passing through A and cutting the mirrors OM_1 and OM_2 at N_1 and N_2 respectively.

The images of A, formed by successive reflection at the mirrors OM_1 and OM_2, must evidently all lie on this circle. Thus, the image A_1', formed by a first reflection at the mirror OM_1, is at a point on the normal ANA_1', such that the distance $A_1'N$ is equal to AN. It follows at once, by *Euc.* i. 4, that the triangles ONA and ONA_1' are equal in all respects, and that, therefore, OA_1' is equal to OA. That is, the point A_1' lies on the circle just described. In the same way it can be shown that all the images formed must lie on this circle. The images formed in this case consist, as already explained in Art. 37, of two series; the series A_1', A_1''. A_1''', ... formed by successive reflections beginning at the mirror OM_1, and the series A_2', A_2'', A_2''', ... formed by successive reflections beginning at the mirror OM_2.

It will be seen from the figure that any two images of the *same order* in these series are, as in the case of parallel mirrors, always on opposite sides of the mirrors, and that the angular distance between any two images of the nth order is always $2n$ times the angle between the mirrors. Thus, if a denote the angle between the mirrors, then the angular distance between the two *first* order images, A_1' and A_2', is $1(2a)$; between the two *second* order images, A_1'' and A_2'', $2(2a)$; between the two *third* order images, A_1''' and A_2''', $3(2a)$; and between two images of the nth order, $n(2a)$ or $2na$.

Now, if the value of a is such that $2na = 360°$, then it is evident that images are formed in each series up to the nth order, given by $n = \frac{360}{2a}$, and that the two images of this order are coincident in the angle vertically opposite the angle between the mirrors.

That is, if a, the angle between the mirrors, is an **even sub-multiple** of 360°, equal to $\frac{360}{2n}$, or **a sub-multiple** of 180°, equal to $\frac{180}{n}$, then images are formed in each series up to the nth order, and the two images of this order are coincident, so that only $(2n-1)$ of the $2n$ images formed are visible as distinct images.

For example, if $a = \frac{360^\circ}{6}$, or 60°, the number of images formed, as shown in Fig. 42, is six, and since the two third-order images, A_1''' and A_2''' are coincident, only five of these are

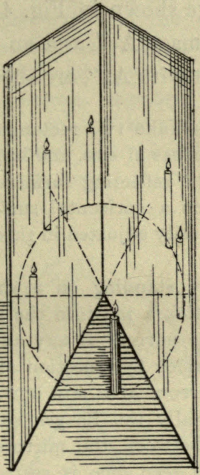

Fig. 41.

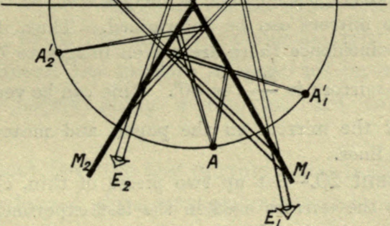

Fig. 42.

visible as distinct images. The five images which can be seen when a candle flame is placed between two plane mirrors inclined at an angle of 60°, are shown in Fig. 41. The paths by which an eye at E_1 sees the image A_1''', and an eye at E_2 sees the image A_2''' which is coincident with A_1''', are shown separately in Fig. 42.

Experiment 19.—Take two rectangular pieces of mirror glass, about 4 inches by 3 inches in size, and hinge them together along two shorter edges, with the reflecting surfaces inwards, so that the pieces will stand securely on a horizontal surface with the hinged edges vertical, and can be easily adjusted in this position to make any angle with each other.

Place these mirrors standing on a sheet of paper pinned on a drawing-board, and stick a pin into the board in the space between the mirrors. Set the mirrors at any angle, and note the circle of images of the pin formed by reflection at their surfaces. Alter the angle between the mirrors, and note how the number of images seen changes. Note specially the images seen in the angle vertically opposite the angle between the mirrors. It will be found that when two images are seen, one is seen by one eye and the other by the other eye (see Art. 39). This is readily verified by closing first one eye and then the other. It will also be found that, as the angle between the mirrors is changed (increased or decreased), the distance between these two images changes in a regular way; if the distance is observed decreasing, it decreases until the images become coincident, then increases to a maximum, and again decreases to coincidence, and so on. The coincidences obviously occur when the angle between the mirrors is an even sub-multiple of 360°, and if at any coincidence the number of images seen be counted, the angle between the mirrors can be determined. Thus, if at a particular position of coincidence there are eleven images to be seen, the angle between the mirrors is $\frac{360°}{12}$ or 30°. This can be verified by marking the trace of the mirrors on the paper, and measuring the angles between the lines.

Experiment 20.—Set up two pieces of thin, clear glass in the same way as the mirrors used in the last experiment on a sheet of paper on a drawing-board, adjust them at an angle of 60° with each other, and between them stick a pin vertically into the board. Let OM_1 and OM_2 (Fig. 42) represent the planes of these mirrors, and A the position of the pin between them.

The images A_1', A_2'', A_2''' (or the coincident image A_1'''), A_1'', and

A_2' can then be seen in the positions shown in the figure. Mark the positions of these images by other pins, as in Experiment 17, and draw lines marking the traces of the mirror surfaces. Now remove pins and mirrors, and prove (1) that the images all lie on a circle whose centre is at O, and radius OA; (2) that the angles AOM_1, $A_1'OM_2$, $A_1''OM_1$, $A_1'''OM_2$, and also the angles AOM_2, $A_2'OM_1$, $A_2''OM_2$, and $A_2'''OM_1$ increase regularly by an angle equal to M_1OM_2; and (3) that the angular distance between two images of the same order is $n(M_1OM_2)$, where n is the order of the images. Draw also the path by which light from A reaches an eye at E as if from the image A_2'''.

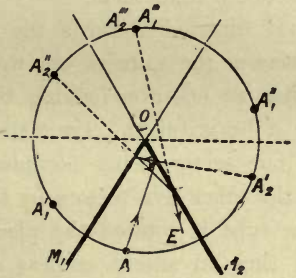

Fig. 43.

41. **The Kaleidoscope.**—When two plane mirrors are inclined to each other at an angle which is an even sub-multiple of 360°, the images of any object placed between the mirrors form, with the object itself, a symmetrical circular pattern. These images lie, as in Fig. 42, in a plane at right angles to the line of intersection of the mirrors, and can all be seen by an eye looking at the object along a line parallel to the line of intersection.

When the object between the mirrors consists of a number of small pieces of coloured glass arranged, more or less, in one plane at right angles to the intersection of the mirrors, the patterns formed show a beautifully symmetrical distribution of form and colour. An instrument in which patterns of this kind can be seen and studied is called a **kaleidoscope.**

The construction of a common form of the instrument is comparatively simple. Two long narrow mirrors inclined at an angle of 60° are mounted in a somewhat longer tube of cardboard. One end of the tube is closed by an opaque disc pierced with a hole through which the observer looks. At the other end a disc of clear glass is first fitted into the tube close up to the ends of the mirrors; this disc is then covered lightly with pieces of coloured glass or other transparent material, and these pieces are kept loosely in position by a disc of ground glass fitted into the tube behind them.

When the tube is directed towards the light, so that the coloured objects between the mirrors are well lighted through the ground glass disc, an observer looking through the hole at the other end sees a beautiful and symmetrical pattern in six sectors of a circle, one sector being occupied by the coloured pieces of glass and the other five sectors by the images of these objects. When the tube is rotated the pieces of glass change their positions, and thus an almost endless variety of patterns may be observed.

In some instruments three long mirror strips, forming a hollow triangular prism of equilateral section, are used instead of two, as here described. The object between the mirrors is now an equilateral triangle covered with small pieces of coloured glass, and as each pair of mirrors gives five images of this triangle grouped in a circle round the lines of intersection of the pair, a somewhat intricate but symmetrically arranged and coloured pattern is formed. The study of these patterns is sometimes of service in the elaboration of ornamental designs.

CHAPTER V.

REFLECTION AT SPHERICAL SURFACES.

42. **Preliminary Definitions.**—A spherical mirror is usually a very small segment of a spherical surface, and may be either concave, as in Fig. 44, or convex, as in Fig. 45.

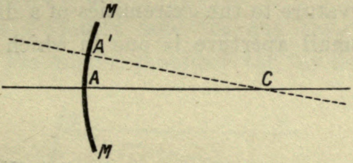

Fig. 44.

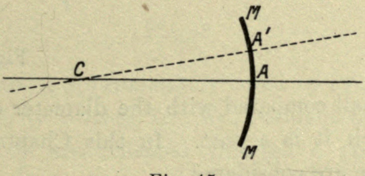

Fig. 45.

In a concave spherical mirror the concave surface of the segment is the reflecting surface, and in a convex mirror the convex surface is the reflecting surface.

The centre C of the spherical surface of which the mirror is a part is called the **centre of curvature** of the mirror.

The point A at the centre of the reflecting surface of the mirror is called the **pole**, or the centre of the face of the mirror.

The straight line passing through C, the centre of curvature of the mirror, and A, the pole of the mirror, is called the **principal axis** of the mirror; any other straight line passing through the centre of curvature C and cutting the mirror at any point, A′, other than the pole, is called a **secondary axis** of the mirror. Since the mirror is a segment of a spherical surface any straight line through the centre of the mirror is **normal** to the mirror; the principal axis or any secondary axis is, therefore, normal to the mirror surface.

A section of the mirror by a plane passing through the pole and the centre of curvature, as shown in all the figures in this Chapter, is called a **principal section.** The **diameter** of the mirror is the diameter MM of its circular edge, and the **aperture** of the mirror is the angle MCM between two lines drawn from the centre of curvature to the extremities of a diameter.

A mirror of **small aperture** is one in which the diameter of the mirror is small compared with the diameter of the spherical surface, of which it is a part. In this Chapter only mirrors of small aperture are considered.

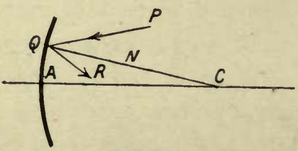

Fig. 46.

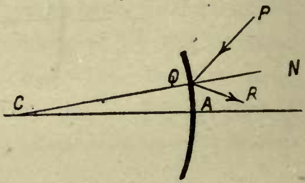

Fig. 47.

43. **Construction for the Reflected Ray.**—Let PQ represent any ray incident at Q on a spherical mirror, concave as in Fig. 46, or convex as in Fig. 47. At Q draw the normal QN to the reflecting surface by joining CQ and producing it if necessary. Then, in accordance with the laws of reflection, the reflected ray QR is obtained by drawing QR in such a direction that the angle of reflection NQR is equal to the angle of incidence PQN.

It is evident from this that a ray incident on the mirror in any direction passing through C is incident normally on the mirror, and is reflected back along the path by which it came.

44. **Principal Focus and Focal Length of a Spherical Mirror.**—Let PQ, Fig. 48, represent a ray incident on a concave mirror in a direction parallel to the principal axis of the mirror. Draw QC the normal to the mirror at the point of incidence Q, then the incident ray will be reflected in a direction, QF, cutting the principal axis in F, and making the angle CQF equal to the angle PQC. But the angle PQC is equal to the angle FCQ (*Euc.* i., 29), therefore the angle FQC is equal to the angle FCQ, and FQ is equal to FC. Now *if* AQ *is small*, FQ is approximately equal to FA, and, therefore, FC is approximately equal to FA, or the point F is midway between the pole, A, and the centre of curvature, C.

This result is evidently true for any ray parallel to the principal axis of the mirror, provided the ray is close enough to the axis for FQ to be approximately equal to FA. Hence, *if the aperture of the mirror is small*, all rays incident on it parallel to the principal axis are reflected through a point, F, on the principal axis, halfway between A, the pole, and C, the centre of curvature of the mirror. This point F is called the **principal focus** of the mirror, and the distance AF, between the pole A and the principal focus F, is the **focal length** of the mirror.

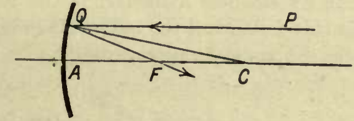

Fig. 48.

Similarly, in the case of rays incident on the mirror in a direction parallel to a secondary axis, the rays are evidently reflected through a point on the secondary axis, halfway between the point where the axis cuts the mirror, and the centre of curvature. This point on a secondary axis is sometimes called a **secondary focus** of the mirror.

If, therefore, a parallel beam of light is incident on a concave mirror, in a direction parallel to the principal axis of the mirror, it will be reflected as a convergent beam through a **real** focus at F the principal focus of the mirror.

Experiment 21.—Let a beam of sunlight fall upon a concave mirror of small aperture, in a direction parallel to the principal axis of the mirror.

Hold a narrow strip of cardboard (narrow enough not to intercept too much of the incident beam) so as to receive the reflected beam. It will be found that the width of the reflected beam, as indicated by the bright patch on the surface of the strip facing the mirror, gradually decreases as the distance of the strip from the mirror increases, and at a certain distance it is reduced to a small intensely bright spot; beyond this point it becomes wider and wider as the distance from the mirror increases. The point at which the reflected beam comes to a focus is the **principal focus** of the mirror, and the distance of the strip from the mirror when the reflected beam is focussed on it to its smallest diameter, is the **focal length** of the mirror. Double this distance is the radius of curvature of the mirror which can thus be determined experimentally.

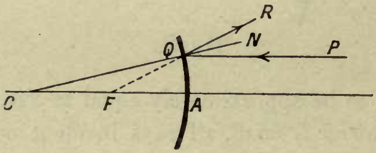

Fig. 49.

In the case of a convex mirror, if PQ, Fig. 49, represent a ray incident on the mirror in a direction parallel to the principal axis, the line QN, drawn from C through Q, is the normal at Q, the point of incidence, and the direction of the reflected ray QR is such, that the angle NQR is equal to the angle PQN. The reflected ray in this case does not, as in the case of a concave mirror, cut the principal axis, but when its direction is produced backwards through Q it cuts the axis at a point F. Now the angle PQN is equal to the angle FCQ (*Euc.* i. 29), and the angle NQR is equal to the angle FQC (*Euc.* i. 15), and since the angle PQN is equal to the angle NQR, the angle FCQ must be equal to the angle FQC, and FC equal to FQ.

If the incident ray is close to the principal axis, so that AQ is very small, the distance FQ is approximately equal to FA, and CF is, therefore, equal to FA. That is, the point F is midway between A, the pole, and C the centre of curvature of the mirror.

Hence, as in the case of a concave mirror, all rays incident on a convex mirror of small aperture, in a direction parallel to the principal axis are reflected in directions which, when produced backwards, pass through the point F on the principal axis, half-way between the pole and the centre of curvature of the mirror; this point is called the **principal focus** of the mirror, and the distance AF, from the pole to the principal focus, is the **focal length** of the mirror; the corresponding point on any secondary axis is a **secondary focus** of the mirror.

Hence, if a parallel beam of light is incident on a convex mirror in a direction parallel to the principal axis of the mirror, it will be reflected as a divergent beam diverging from a **virtual** focus behind the mirror at F, the principal focus of the mirror.

Experiment 22.—Arrange an experiment exactly similar to Experiment 21 with a convex mirror of small aperture.

It will be found that the diameter of the reflected beam, as shown by the length of the brightly illuminated portion of the strip of cardboard used as a screen, increases as the distance of the screen from the mirror increases. This shows that the reflected beam is a divergent beam diverging from a point behind the mirror.

Find the position of the screen at which the diameter of the divergent beam is exactly twice the diameter of the mirror, and then measure the distance of the screen from the mirror. Draw a diagram showing that this distance is equal to the distance of the focus of the reflected beam behind the mirror, and so gives **indirectly** a measure of the focal length of the mirror.

It should be noted that the focus of the reflected beam, being **virtual**, cannot be shown on the screen.

45. **Reflection of a Divergent or Convergent Pencil of Rays at the Surface of a Spherical Mirror.**—It can be shown that, when a pencil of rays diverging from, or converging to, a point on the principal axis of a spherical mirror is reflected at the surface of the mirror, the reflected pencil also diverges from, or converges to, a point on the principal axis. Thus, in Fig. 50, let PQ represent *any one ray* of a pencil of rays diverging from the point P on the principal axis of a concave mirror. From C, the centre of curvature of the mirror, draw CQ, the normal to the mirror at Q, the point of incidence of the ray. The reflected ray QP′ can now be drawn by making the angle of reflection CQP′ equal to the angle of incidence PQC. Let the reflected ray QP′ cut the principal axis at the point P′.

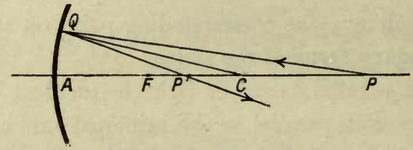

Fig. 50.

Then, in the triangle PQP′, the vertical angle PQP′ is bisected by the line QC cutting the base PP′ at C, and, therefore (*Euc.* vi. 3),

$$\frac{QP'}{QP} = \frac{P'C}{CP}.$$

But, *if* AQ *is small*, then QP′ and QP are approximately equal to AP′ and AP respectively, and, therefore,

$$\frac{AP'}{AP} = \frac{P'C}{CP}.$$

That is,

$$\frac{AP'}{P'C} = \frac{AP}{CP}.$$

This relation implies that the position of the point P′ is fixed by the position of the point P,* and, as this is true for *any ray* of the pencil, it follows that all rays of a *small pencil* diverging from P are reflected from the mirror through the same point P′. That is, P′ is the focus of the reflected pencil.

This proof deals with only one case of reflection at a spherical mirror, but it can be shown, in a similar way for all cases, that, when a *small pencil* of rays diverging from, or converging to, any point on the principal axis of a spherical mirror is reflected at the surface of the mirror, the reflected pencil diverges from, or converges to, another point, also on the principal axis. Similarly, if the focus of the incident pencil is a point on a secondary axis of the mirror, then the focus of the reflected pencil is another point on the same secondary axis, and the relation defining the relative position of the two points is the same as for points on the principal axis.

46. **Conjugate Foci.**—It has been shown, in the preceding article, that the image of a point P, on the principal axis of a spherical mirror, is formed at a point P′, also on the principal axis. That is, if the point P on the principal axis be the focus of a pencil of light incident on the mirror, then the point P′, also on the principal axis, is the focus of the pencil after reflection at the surface of the mirror.

It is obvious, from the construction shown in Fig. 50, that the two foci, P and P′, are interchangeable; that is, if P′ is the focus of the reflected light when P is the focus of the incident light, then P would be the focus of the reflected light if P′ were the focus of the incident light.

This interchangeable, or **conjugate**, relation between the foci is usually expressed by saying that the foci P and P′ are **conjugate foci.** That is, if any two points on the principal axis of a spherical mirror are related as foci of an incident pencil of light and the corresponding reflected pencil, or as **object** and **image**, by reflection at the surface of the mirror, then this relation is an interchangeable or conjugate one, and the two points are called **conjugate foci.** Any two points related as object and image by reflection at the surface of a spherical mirror are, strictly, conjugate foci, and lie on an axis of the mirror, but the term is generally applied only to points on the principal axis.

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* The point P′ evidently divides AC, the radius of curvature of the mirror, *internally* in the same ratio as P divides it *externally*.

If the conjugate foci, P and P′, be considered as object and image respectively, the position of P′ is determined by the position of P, and can be specified for any given position of this point. The general cases which present themselves in considering the relative positions of these two points are indicated, for a concave mirror, in Figs. 51, 52, and 53, and for a convex mirror in Fig. 54.

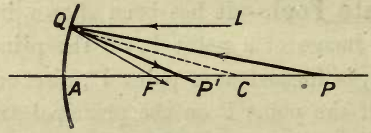

Fig. 51.

In Figs. 51, 52, and 53 the ray LQ, incident on the mirror in a direction parallel to the principal axis, is reflected through the principal focus F, in accordance with the definition of this point. Hence, if any ray of light, PQ, is incident on the mirror, as shown in Fig. 51, at the point Q, from any point P beyond C on the principal axis—that is, from *any point* on the principal axis at a distance from the mirror greater than the radius of curvature—the angle of incidence PQC must be less than the angle LQC; the angle of reflection CQP′ is, therefore, less than the angle CQF, and the reflected ray cuts the principal axis at a point between F and C.

Hence, if the point P is anywhere on the principal axis beyond the centre of curvature, C, the conjugate focus P′ is somewhere between F and C on the principal axis. In this case the focus of the reflected pencil at P′ is obviously **real.**

It follows at once from this, by reversal of the path of the light between P and P′, that if, as in Fig. 52, P is anywhere between C and F, the conjugate focus P′ must lie somewhere beyond C. In this case, also, the focus of the reflected pencil of light is **real.**

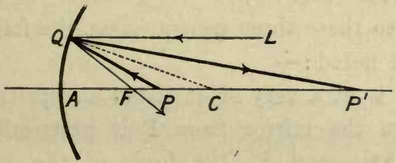

Fig. 52.

If, however, P is anywhere between F and A, as in Fig. 53—that is, at a distance from the mirror less than its focal length—then PQC, the angle of incidence of the ray PQ, is greater than the angle FQC, and the angle of reflection CQR is, therefore, greater than the angle CQL; the pencil of light reflected from the mirror therefore diverges as if from a point, P′, behind the mirror.

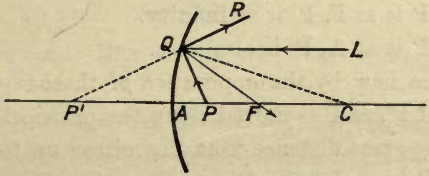

Fig. 53.

Hence, if the point P is anywhere on the principal axis between F and A, the conjugate focus P′ is **virtual,** and lies somewhere on the principal axis behind the mirror.

Thus, in the case of a concave mirror there are three general cases for which a general specification of the relative positions of the conjugate foci can be given. These three cases may be stated as follows:—

(1) When P is beyond C, P′ is real, and lies between F and C (Fig. 51).

(2) When P is between C and F, P′ is real, and lies beyond C (Fig. 52).

(3) When P is between F and A, P′ is virtual, and lies behind the mirror (Fig. 53).

In addition to these three general cases, the following special cases should be noted:—

(4) When P is at a very great distance from the mirror, any ray incident on the mirror from P is practically parallel to the principal axis, and P′, the focus of the reflected light, would therefore be at F.

That is, when P is at infinity, P′ is at F.

(5) When P is at C any ray incident on the mirror from P is normal to the mirror and is, therefore, reflected back through P, so that if a pencil of light from P is reflected at the mirror, the focus of the reflected pencil is also at P.

That is, if P is at C, P′ is also at C.

(6) When P is at F, P′ is at infinity.

(7) When P is at A, P′ is also at A.

It is possible now by the application of these results to trace the position of P′, as P is moved along the principal axis from a point at a very great distance from the mirror up to the mirror. Thus, when P is at infinity P′ is at F, and as P is moved from infinity up to C, P′ moves to meet P from F to C. The points P and P′, therefore, meet at C and are coincident at this point as explained above. Then as P is moved on from C to F, P′ obviously moves on in the opposite direction from C to infinity, and as P is moved still further from F to A, P′ moves from a point at an infinite distance behind the mirror, up to the mirror where P and P′ are again coincident.

In tracing the corresponding motions of P and P′ in this way, it is helpful to remember that the points being related as object and image by reflection, always move along the principal axis in **opposite directions.** This is always the case for an image produced by reflection at any mirror; for example, if a man standing in front of a mirror moves towards it his image moves to meet him, and if he walks away from the mirror his image moves away in the opposite direction.

It is also to be noted that if, while P is moving in a given direction along the axis, P′ moves off to a point at an infinite distance in front of the mirror, it moves back from a point at an infinite distance behind the mirror. For example, while P moves through F towards the mirror, P′ disappears to infinity in front of the mirror and then reappears from infinity behind the mirror.

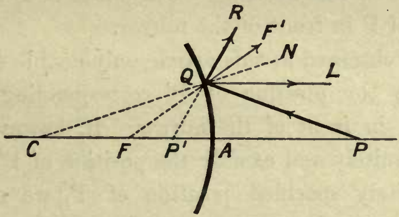

Fig. 54.

In the case of a convex mirror, Fig. 54, the ray LQ incident on the mirror at Q, in a direction parallel to the principal axis is reflected in a direction QF′ passing through F, the principal focus of the mirror.

It will be seen from the figure that if the point P is on the principal axis anywhere in front of the mirror, the angle of incidence PQN, for a ray PQ incident on the mirror at Q, is *greater than* the angle LQN; the angle of reflection NQR is, therefore, greater than the angle NQF′, and the reflected pencil diverges from a point P′ between F and A on the principal axis behind the mirror.

Hence, in the case of a convex mirror if the point P is on the principal axis anywhere in front of the mirror, the conjugate focus P′ is a **virtual** focus, and lies on the principal axis behind the mirror between the pole of the mirror A and the principal focus F.

This is the only general result necessary, in the case of a convex mirror, for the general specification of the position of P′ as the focus conjugate to P. It will also be obvious from the figure that when P is at an infinite distance in point of the mirror, P′ is at F behind the mirror and is virtual. Hence, when P moves from a point at an infinite distance in front of the mirror up to the mirror at A, P′ moves from F behind the mirror up to A where the two foci coincide. The focus P′ is thus virtual for all positions of P in front of the mirror.

The results obtained in this article only enable us to specify *in a general way* the position of P′ corresponding to any given position of P in front of the mirror. If, however, we wish to determine definitely and exactly the position of P′ corresponding to any definitely specified position of P, we must obtain a quantitative relation between the distances of the points P and P′ from some definite point on the principal axis. This quantitative relation is given in Art. 50.

In this simple discussion of conjugate foci the point P is assumed to be always in front of the mirror, as it would be if it were a luminous point or a point on any object. If, however, as in Art. 32, for a plane mirror, the pencil of light incident on the mirror is a convergent pencil, then P, as the focus of the incident pencil of light, may be at any point behind the mirror, and it becomes of interest to trace the position of P′ as P, the virtual focus of the incident pencil, moves from A to a point at an infinite distance behind the mirror. The student will find it a good exercise to follow the method of this article and trace for both a concave and a convex mirror the positions of P′ for all positions of P behind the mirror.

47. General Construction for Determining the Image of an Object Formed by Reflection at a Spherical Surface.

—The general construction for determining the image of an object formed by reflection, at the surface of any mirror, has been explained in Art. 33. It will be clear, however, from what has been said in later articles, that in the case of a spherical mirror this general construction is simplified by the fact that, from any point on the object, three incident rays may be drawn for which the directions of the corresponding reflected rays are known. Thus, in Fig. 55, the ray PL, incident parallel to the principal axis, is reflected through the principal focus F; the ray PN, incident in a direction passing through the centre of curvature C, and normal to the mirror, is reflected back along the normal in a direction passing through C; and the ray PM, incident in a direction passing through the principal focus F, is reflected in a direction parallel to the principal axis. Hence, in applying the general construction to determine the image of any point on the object, any two of these three incident rays may be drawn from the point, and there are therefore three geometrical constructions available for the determination of the image of an object. These constructions are shown in the three figures given below. For the purposes of comparison the figures are drawn to represent the same case of reflection at the surface of a concave mirror. The different cases that occur for different positions of the object are dealt with later.

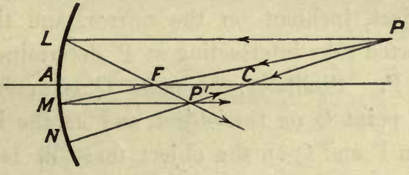

Fig. 55.

In these figures (Figs. 56-58) PQ represents an object placed in front of a concave mirror, at a point beyond the centre of curvature of the mirror, and P′Q′ represents the image of this object as determined by the constructions shown in the figures. For clearness the construction lines for determining the image of the point P only are shown in the figures; the lines for the point Q, or for any other point on the object, correspond exactly to those shown for P.

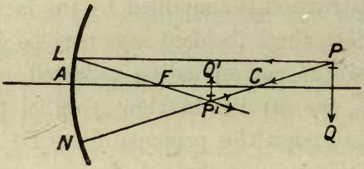

Fig 56.

In Fig. 56 the two rays PL and PN are drawn from the point P on the object, incident on the mirror, and the two corresponding reflected rays intersecting at P′ determine this point as the image of P. Similarly, the point Q′ is determined as the image of the point Q on the object, and as the images of the points between P and Q on the object must lie between P′ and Q′, the image of the object PQ is completely determined at P′Q′. In the case of an object of complicated form it may, of course, be necessary to determine the image of more than two points on the object, in order to determine completely the image of the object as a whole.

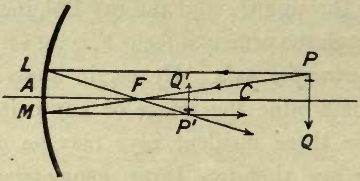

Fig. 57.

In Fig. 57 the two rays PL and PM are drawn incident on the mirror from the point P on the object, and the two corresponding reflected rays intersecting at P′ determine this point as the image of P. Similarly, the point Q′ is determined as the image of the point Q, and P′Q′ as the image of the object PQ.

In Fig. 58 the two rays PM and PN are drawn from the point P on the object, and the two corresponding reflected rays, intersecting at P′, determine this point as the image of P. Similarly, the point Q′ is determined as the image of the point Q, and P′Q′ as the image of the object PQ.

The most usual of these three constructions is the one shown in Fig. 56; occasionally, however, it is necessary or convenient to use one or other of the two remaining constructions.

48. **Relative Position of Image and Object.**—It will be noticed that any point on the object and the corresponding point on the image are really conjugate foci, so that the point of the object which lies on the principal axis and the corresponding point on the image, which must also be on the principal axis, are conjugate foci as defined in Art. 46. It follows from this that the general cases given in Art. 46 with reference to the relative positions of conjugate foci, apply also to the relative positions of object and image, and enable us to specify, in general terms, the position and character of the image for any given position of the object in front of a spherical mirror.

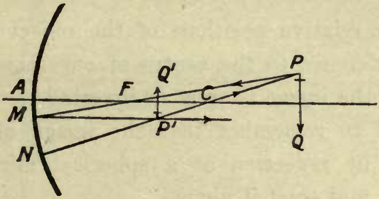

Fig. 58.

In order to specify the character of an image it is necessary to state—

(1) Whether the image is **real** or **virtual.** This depends upon whether the light from the object after reflection at the

surface of the mirror actually passes through the image or only appears to diverge from it.

Whenever the image appears to be behind the mirror it must necessarily be virtual. It should be remembered, too, that a real image can, from its character, be focussed on a screen, whereas a virtual image, being only subjective and not real, cannot be thrown on a screen at all.

(2) Whether the image is **erect** or **inverted.** This can readily be determined by drawing the construction for the image in any given case. It is evident, however, that from Figs. 56-58 that when object and image are on opposite sides of C, the centre of curvature, the image must be inverted on account of the crossing of the rays passing through C from the top and bottom of the object.

Hence, if the relative positions of the object and image are known, with reference to the centre of curvature, it is easy to decide whether the image is erect or inverted.

It is helpful to remember that the image of a real object (when formed by reflection at a spherical mirror) is always inverted if real, and erect if virtual.

(3) Whether the image is **magnified** or **diminished** in size compared with the object. This is usually expressed by giving the ratio of corresponding **linear** dimensions of the image and object. This ratio, generally expressed as $\frac{image}{object}$, is called the **magnification**, whether the object be magnified or diminished, and is usually denoted by m. If m denote the linear magnification of the image, then m^2 obviously denotes the area or surface magnification.

The different cases which arise in discussing the relative positions of image and object for reflection at the surface of a spherical mirror, are illustrated by the figures given below. These figures refer, respectively, to the different cases given in detail for conjugate foci in Art. 46.

Taking first the cases which relate to reflection at the surface of a **concave mirror**, we get the following six cases:—

(1) *When the object is at infinity, or at a very great distance from the mirror, the rays incident on the mirror from any point on it are practically parallel.*—Hence, in Fig. 59, if the object PQ is very distant from the mirror, a pencil of rays from the point P at the top of the object is incident on the mirror as a parallel pencil, and, if the axis of this pencil be drawn through C, the rays will be reflected through a point, P′, a secondary focus of the mirror on the secondary axis PC. Similarly, rays from the point Q at the bottom of the object are incident on the mirror as a parallel pencil, and if a pencil be taken, as in the figure, with its axis passing through C, the rays will be reflected through the secondary focus Q′ on the secondary axis QC. An image of PQ is thus determined at P′Q′ at a distance from the mirror, practically equal to its focal length, for the image of any point of the object on the principal axis will obviously be at F, the principal focus of the mirror. The image at P′Q′ is obviously **real**, **inverted**, and **diminished**. It should be noted that although rays from P are parallel to each other and rays from Q are parallel to each other, a ray from P is not parallel to a ray from Q; the angle between the rays from P and the rays from Q is obviously the angle PCQ, which the object PQ subtends at the centre of the mirror. If the object is a very large one it may, although at a very great distance from the mirror, subtend an appreciable angle at C. Thus the sun, although at a distance of over 92,000,000 miles from the earth,

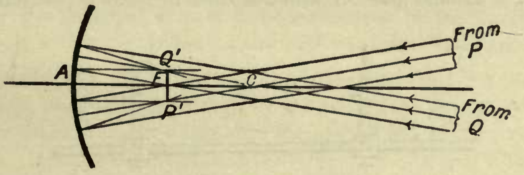

Fig. 59.

subtends an angle of about 32′ at the surface of the earth. The image of the sun would, therefore, be a disc of diameter P′Q′ (Fig. 59) subtending an angle of 32′ (P′CQ′) at a distance, CF, equal to the focal length of the mirror.

If the object is so distant that the angle PCQ is extremely small, the image P′Q′ is reduced practically to a point. Thus fixed stars are so distant from the earth that, although some of them are much greater than the sun, the angles which their diameters subtend at the surface of the earth are inappreciably small and their images would, therefore, be mere points.

Experiment 23.—Take a concave mirror of long focal length, mounted on a stand, and adjust its position so that it faces the sun with its principal axis directed towards the sun. Hold a small screen of ground glass or asbestos card in front of the mirror, as in Exp. 21, so as to receive the reflected beam of light, and adjust its position until a clearly-defined disc, the image of the sun, is focussed on the screen. Measure, along the principal axis, the distance of the screen from the pole of the mirror. This distance gives the focal length of the mirror, and twice this distance gives the radius of curvature. If possible, measure also the diameter of the image of the sun, and draw the triangle P′Q′C (Fig. 59) to scale; then measure the angle P′CQ′. This is the angle which the sun subtends at the surface of the earth.

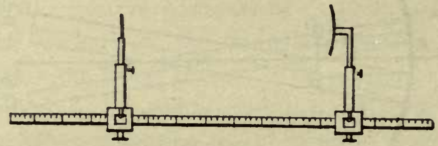

Fig. 60.

Notice the heating effect of the rays concentrated at the image of the sun; if the mirror is a good one, a large percentage (Art. 26) of the direct sunlight that falls on the surface of the mirror is here focussed on the much smaller surface covered by the sun's image, and the heating effect of the sun's rays is, therefore, proportionately increased.

For the purpose of this and similar experiments, the mirror and the screen should be carried by stands made to slide along a light graduated bar, as shown in Fig. 60.

The mirror is mounted with its principal axis parallel to the bar, and the plane of the screen is at right angles to the principal axis. The bar is directed towards the distant object whose image is required, and the position of the stand carrying the screen is adjusted until the image is sharply focussed on the screen. The distance between the screen and the mirror is then read off on the graduated bar by means of index marks on the slide carriers marking the position of the pole of the mirror and the plane of the screen surface. With an ungraduated bar the distance between the pole of the mirror and the surface of the screen may be accurately measured by means of callipers or an extending rod with pointed ends.

The dimensions of the screen should be small compared with the diameter of the mirror, so as to intercept as little as possible of the incident beam. If necessary a larger screen may be used by arranging the experiment so that the object whose image is to be thrown on the screen is a little above or a little on one side of the principal axis; the image will then be formed a little below or a little on the other side of the principal axis, and the screen can be put in position to receive it without intercepting the incident light to any great extent.

Experiment 24.—Take the apparatus described for the last experiment, and focus the image of a distant object, such as a church spire, a chimney stack, a tree, or a distant window in the room, on the screen. Measure the distance of the screen from the pole of the mirror. It will be found that the distance is practically the same for all distant objects, and equal to the focal length of the mirror. This is the most convenient method of finding the focal length of a concave mirror.

If the object chosen is not distant enough compared with the focal length of the mirror to be considered a very distant object, then it will be found that the distance from the screen to the mirror is slightly greater than the focal length. Try different objects, and verify that, as the object gets nearer the mirror, the image moves from the focus towards the centre of curvature.

(2) *When the object is beyond the centre of curvature—that is, at a distance from the mirror greater than the radius of curvature—the image lies between the principal focus and the centre of curvature.*—This is the case illustrated by Figs. 56, 57, and 58, which are explained in detail above. In this case again the image is **real, inverted,** and **diminished.**

Experiment 25.—Take a concave mirror mounted on a stand, and find its focal length roughly, as in Exp. 24, by focussing the image of a distant window on a small card, and measuring the distance of the image from the mirror.

Place the mirror on a table in a darkened room, and adjust the position of the mirror so that the principal axis is directed horizontally along the length of the table. Set a lighted candle on the table at a distance greater than the radius of curvature in front of the mirror, and adjust its position so that the base of the flame is on a level with the principal axis, and about an inch further from (or nearer to) the observer than the axis. Now, take a piece of thin cardboard about the size of a postcard, and mount it on a stand in a vertical plane, so that its middle point is on a level with the principal axis. This card serves as an efficient screen; a piece of thin ground glass or a sheet of paper, or tracing paper, mounted in a frame also gives a very satisfactory screen, and has the advantage that the image can be seen from either side. Place the screen at right angles to the axis, a little nearer to (or further from) the observer than the axis, and move it up to the mirror until an image of the candle flame is seen on the face of the screen next the mirror. The image will probably, when first recognised, be somewhat blurred; focus it clearly and sharply by adjusting the position of the screen nearer to, or further from, the mirror as may be necessary. Notice that the image is **real** (since it can be thrown on a screen), **inverted**, and **diminished**.

Focus the image on the screen for a number of different positions of the candle flame. In any of these positions interchange the object and the screen, and note that a clearly-focussed image appears on the screen in its new position, showing that the positions of object and image are interchangeable as **conjugate foci**. Find out in what direction the image moves along the axis as the flame is moved from a point at a good distance (say twice the radius of curvature) from the mirror up nearly to the centre of curvature. Note also how the **magnification** of the image changes as its position changes.

(3) *When the object is at the centre of curvature, the image is also at the centre of curvature.*—This case is illustrated by Fig. 61, where PC represents the object, and P′C the image. The construction for this image should be noted. Any two rays incident on the mirror from C, such as CN and CN′, are normal to the mirror, and are, therefore, reflected back through C where they intersect—that is, the image of C is formed at C. The image of P is determined at P′ by the usual construction, in which the two rays incident from P on the mirror, PL and PM,

are drawn respectively parallel to the principal axis, and through the principal focus; the corresponding reflected rays LP′ and MP′ intersect at P′, and determine the image of P at this point. It is obvious from the geometry of the figure that the image P′C is equal to the object PC. In this figure the object is shown on one side of the principal axis, not bisected by the axis as in other similar figures; this is merely for clearness, if an object PQ were taken with its middle point on the principal axis, as in Fig. 56, it will be evident from the construction that P′, the image of P, would be formed at Q, and Q′, the image of Q, would be formed at P, and the inverted image P′Q′ would coincide exactly with the object QP, P′ coinciding with Q, and Q′ with P.

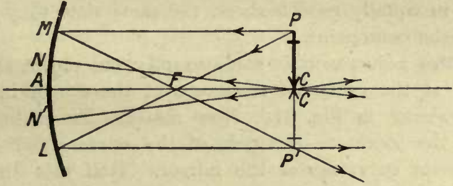

Fig. 61.

In this case, then, the image is **real, inverted**, and **equal** to the object.

Experiment 26.—Take a concave mirror mounted on a stand and adjust its position so that the principal axis is horizontal. Get a large needle, or a pointed piece of white wood, or a piece of stiff white paper cut in the form of an arrow point, or some such object, and mount it in a vertical position, point upwards, so that the point is on a level with the principal axis.

Now set this object in front of the mirror with its point on the principal axis, and arrange the apparatus so that the side of the object facing the mirror is well lighted; the apparatus may, for example, be arranged on a table with the back of the mirror facing a window. If the paper point is used as an object the plane of the paper should be at right angles to the principal axis.

When these adjustments are made, look along the principal axis at the mirror from a point some distance further from the mirror than the object set up in front of it. A real inverted image of the object will generally be seen in mid air. If no image can be seen the object is probably too near the mirror; if it stands between the principal focus and the mirror, an erect virtual image should be seen in the mirror; if it stands between the focus and the centre of curvature, but too near the focus, a real image is formed, probably behind the observer. A real inverted image can, however, always be brought into view by moving the object along the axis. When this image is found, focus the eyes on it until it is distinctly seen and note whether it is nearer or further away than the object, then move the object along the principal axis towards the image so as to bring the point of the object and point of the image into exact coincidence. The final adjustment to exact coincidence is made, as in Exp. 17, by adjusting until the points do not appear to alter their relative position when the eye is moved from side to side. A more exact adjustment is made by viewing the point of the object and the point of the image with a magnifying glass; if they are both in equally good focus at the same time they must be very nearly at the same point.

When this adjustment is made exactly, the object and image are obviously at the centre of curvature of the mirror in the relative positions shown in Fig. 61. Now measure the distance from the point of the object to the pole of the mirror, and so determine the radius of curvature of the mirror. Half this distance is the focal length of the mirror.

This method of determining the radius of curvature, and, therefore, the focal length of a concave mirror, should be noted particularly; it is the most accurate method of making the determination.

An image formed in this way in mid air in front of the mirror is a real image. Notice that it differs from a real object by not being visible from all points; it can be seen only within the limits determined by the rays involved in its formation.

(4) *When the object is between the centre of curvature of the mirror and the principal focus, the image is beyond the centre of curvature and lies somewhere between that point and infinity.*—This case is illustrated by Fig. 56, if we take P′Q′ as the object and PQ the image of the object. The student should draw a special figure for himself.

In this case the image is **real, inverted,** or **magnified.**

Experiment 27.—Arrange a concave mirror, a candle flame, and a screen as explained in Exp. 25. Determine roughly the focal length of the mirror and then place the candle flame between the focus and the centre of curvature of the mirror, and the screen some distance beyond the centre of curvature. In this case the screen will not come between the flame and the mirror, so both flame and screen may be placed centrally on the principal axis. The flame must not be placed too near the focus, for if it is the image will be formed at too great a distance beyond the centre of curvature. Hence, to begin with, place the flame about half way between the focus and the centre of curvature, and adjust the position of the screen until a sharply defined image of the flame is focussed on it. Note that the image lies beyond the centre of curvature, and is real, inverted, and magnified. Interchange object and screen and note that a clearly focussed image appears on the screen, showing that the two positions are interchangeable as conjugate foci. Make this adjustment for a number of different positions of the candle flame, and note how the image moves as the flame is moved from the centre of curvature up towards the focus. Note, also, how the magnification of the image changes as its position changes.

Experiment 28.—Take a concave mirror and prepare two mounted needles, or paper arrow points, as described in Exp. 26. Set up one of the needles as an object in front of the mirror with its point on the principal axis. Stand back some distance from the mirror and look along the principal axis, as in Exp. 26; if necessary, adjust the position of the needle until a real inverted image of it is seen at some point on the axis. Focus the eyes on this image, take the second mounted needle and adjust it in position, as explained above, so that its point coincides with the point of the image. The first needle point now marks the position of the object and the second needle point the position of the image. The relative position of image and object for different positions of the object can thus be studied experimentally, without the use of a candle flame and screen in a darkened room, as in Exps. 25 and 27.

If one observer at some distance from the mirror looks along the principal axis of the mirror, while another observer moves the needle used as an object along the principal axis, the corresponding motion of the image, as described in Art. 46, can be clearly seen. Notice particularly that object and image always move in opposite directions along the axis. Notice, also, that in general the corresponding displacements of object and image are very unequal; when one moves between infinity and the centre of curvature the other moves between the principal focus and the centre of curvature, hence a small displacement in the latter range must correspond to a much larger displacement in the former. As the object is moved towards the mirror and approaches the principal focus the distance of the image from the mirror increases very rapidly, and ultimately the distance is too great for the image to be observed. If the object is moved past the focus towards the mirror a virtual, erect, and magnified image is seen in the mirror. This case is considered below.

(5) *When the object is at the principal focus the image is at infinity.*—This case is illustrated by Fig. 59, if P′Q′ be taken to represent the object—or a special figure may be drawn in the usual way.

It will be noticed that a pencil of light from any point on the object is reflected as a parallel pencil of light, parallel to the axis of the mirror on which lies the point on the object from which the incident pencil diverges. Hence, if a small source of light be placed, as in Fig. 62, at the principal focus of the mirror, the whole of the light incident on the mirror from the source is reflected as a beam of light parallel to the principal axis of the mirror. This explains the action of the mirror as a *reflector ;* the light incident on the mirror from the source, instead of spreading out in all directions into the surrounding medium and diminishing in intensity in accordance with the law of inverse squares, is reflected from the mirror as a parallel beam, which, since it does not spread out into the medium, remains of constant intensity, except in so far as it may

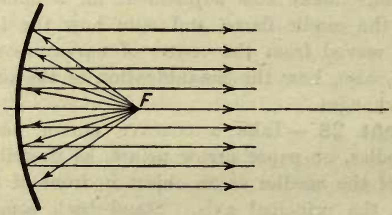

Fig. 62.

be diminished by absorption in the medium through which it travels.

Experiment 29.—Take a large concave mirror and find its focal length by one of the methods described above. Set it up on a table in a dark room and place a small source of light (a small gas flame or candle flame) at the principal focus; then place a large screen so as to receive the reflected light from the mirror.

Notice that the section of the reflected beam, as indicated by the circular bright patch on the screen, is a circle of the same diameter as the mirror. Measure the diameter of the beam at different distances from the mirror; note that it remains constant, indicating that the beam is a parallel beam.

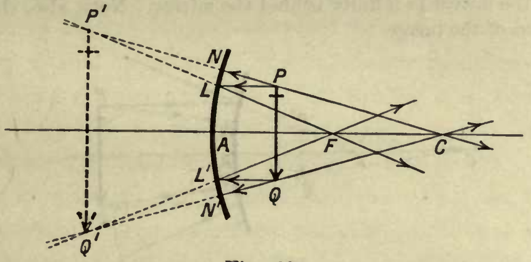

Fig. 63.

(6) *When the object is between the principal focus and the mirror, the image is virtual and lies behind the mirror.*—This case is illustrated by Fig. 63; PQ represents the object, and P′Q′ the image. The construction is the usual construction described above. It will be seen that the image is **virtual, erect,** and **magnified.**

If a small source of light is placed on the principal axis, between the focus and the mirror, the light incident on the mirror will be reflected as a divergent beam diverging from a point behind the mirror. Hence, in the use of a concave mirror as a **reflector,** if the source of light be placed a little nearer to the mirror than the focus, the reflected beam becomes slightly divergent and is more useful for purposes of illumination than a narrow parallel beam.

Experiment 30.—Set up a concave mirror on a table with its axis horizontal. Fix a white knitting pin (a piece of thermometer tubing or some such object) vertically in front of the mirror at a point on the principal axis between the focus and the mirror. Adjust the position of the pin until the virtual image seen in the mirror is at a convenient distance behind the mirror. Then take a second knitting pin, similar to the one taken as object, and adjust its position behind the mirror until the portion seen over the top of the mirror appears as a continuation of the image, at exactly the same distance behind the mirror as the image itself.

The relative position of object and image can thus be studied for different positions of the object between the focus and the mirror. Note how the image moves as the object is moved, and how a small displacement of the object on the short range from the focus to the mirror corresponds to a much larger displacement on the long range, from the mirror to infinity behind the mirror. Note, also, the magnification of the image.

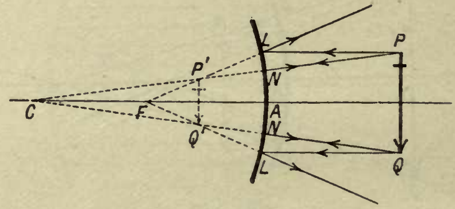

Fig. 64.

In dealing with reflection at the surface of a **convex mirror,** only one case need be considered.

When the object is anywhere in front of the mirror, the image is virtual and lies behind the mirror, between the principal focus and the mirror.—This case is illustrated by Fig. 64, in which PQ represents the object and P′Q′ the image. The construction is the usual general construction already described. The image is obviously **virtual, erect, and diminished.**

Experiment 31.—Set up a convex mirror, as in the last experiment, and place a white knitting pin vertically in front of it, at any point on the principal axis. Fix the position of the virtual image behind the mirror by means of another pin, as already described, and note the relative positions of object and image. Note, also, how the image moves as the object is moved along the axis, and compare corresponding displacements of objects and image. It will be seen that the image is always virtual, erect, and diminished.

49. **The Optical Bench.**—Optical experiments of the type described in the foregoing article are most conveniently performed with the aid of a special piece of apparatus known as an **optical bench.** When the experiments are of a quantitative nature, and exact measurements have to be made, as in experiments to be described later, an optical bench is necessary for exact work.

The general form of an optical bench suitable for elementary work consists of a massive base board of well seasoned wood, about three metres long, constructed to carry stands for the necessary optical apparatus. The stands are mounted on small bases or sliders, which are made either to slide in a groove cut in the base board, or to ride on a rectangular rail or guide bar mounted on the base board, parallel to its length. A scale showing centimetres and millimetres runs along the length of the base board, and is so placed that the position of any slider on the bench can be read on the scale by means of an index line marked on the edge of the slider.

The stands are of various kinds, and are constructed to carry mirrors, lenses, screens, and "objects" of various kinds; they are conveniently made with telescopic slides, so that the different pieces of optical apparatus used in any experiment may be all adjustable to the same level.

Fig. 65 shows a good form of optical bench. The sliders run in grooves cut in the base board, and are so constructed that they do not prevent adjacent stands from being brought quite close together, if necessary. Some of the stands in common use for optical experiments are also shown in the figure.

The optical bench is used chiefly for the experimental determination of the constants of mirrors and lenses. Fig. 65, for example, shows a concave mirror mounted on a stand, M, a candle in another stand, C, and a screen, of thin white unglazed paper in a frame, on a third stand, S. As shown in the figure the position of the screen is so adjusted that an image of the candle flame is focussed on the screen. For any adjustment of this kind the respective distances of the object and image from the mirror, or from any other point, can thus be accurately and conveniently read off on the scale of the bench.

Photometric measurements with Bunsen's or Joly's photometer are also conveniently made on an optical bench; the photometer, mounted on an upright placed between two other uprights carrying the lights to be compared, is readily adjusted in the right position, and the required distances are at once read off on the scale. In cases where an ordinary object, illuminated by daylight, is used as an "object," the experiments must obviously be made in daylight, but for experiments such as Exp. 25 or 27 above, where a candle flame is used, a dark room or, at least, a partially darkened room is necessary. A candle flame is a convenient object for qualitative experiments, but it is too large and not well enough defined for use in experiments involving accurate measurement. A convenient "object" for accurate work is furnished by a sheet of fine wire gauze fixed over a hole in a thin metal plate, and illuminated by a strong light placed behind the plate. A simpler and, in some respects, a more satisfactory

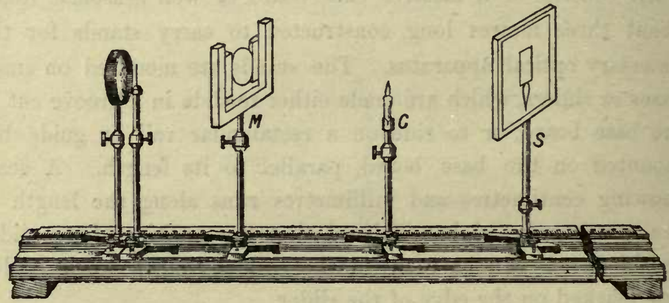

Fig. 65

arrangement is a small screen of tracing paper with four concentric circles, respectively, 10 mm., 15 mm., 20 mm., and 25 mm. in diameter drawn on it, and illuminated by a strong light placed behind the screen. This object is very convenient for use in experiments where measurements of the magnification of the image have to be made.

A small screen of transparent paper or thin ground glass ruled in small squares also forms, when brightly illuminated, a satisfactory object.

For experiments with mirrors the object screen and the image screen should be narrow horizontal or vertical strips; in this form they give no trouble by intercepting the incident or reflected light.

50. Relation Between the Distances of the Object and Image from a Spherical Mirror and the Focal Length of the Mirror.—A simple formula connecting these three distances, and applicable to all cases of reflection at the surface of a spherical mirror, may be obtained by the following geometrical method.

Consider first the general cases of reflection at the surface of a **concave mirror.**

The case in which the object PQ is placed at a distance from the mirror greater than the focal length, and the image P′Q′ is real, is represented in Fig. 66. The construction shown in the figure differs a little from that adopted in Art. 48. The ray QL, parallel to the principal axis, is reflected through F as usual; the ray QA, incident on the mirror at the pole A, is reflected in the direction AQ′, making the angle of reflection P′AQ′ equal to the angle of incidence PAQ. The point Q′, at the intersection of these two reflected rays, is thus determined as the image of Q.

In the figure the triangle APQ and AP′Q′ are obviously similar, and therefore—

$$\frac{PQ}{P'Q'} = \frac{AP}{AP'}.$$

If the aperture of the mirror is small the arc AL may be taken as a straight line, and as the triangles FAL and FP′Q are similar, we have—

$$\frac{\mathrm{AL}}{\mathrm{P'Q'}} = \frac{\mathrm{AF}}{\mathrm{P'F}}.$$

But AL is evidently equal to PQ, and therefore—

$$\frac{\mathrm{AP}}{\mathrm{AP'}} = \frac{\mathrm{AF}}{\mathrm{P'F}}.$$

Now let a, b, and c denote respectively the numbers measuring the lengths of AP, the distance of the object from the mirror, AP′, the distance of the image from the mirror, and AF, the focal length of the mirror. That is, a, b, and c are merely numbers measuring these distances in any convenient unit *without consideration of direction.* Then we have $\mathrm{AP} = a$, $\mathrm{AP'} = b$, $\mathrm{AF} = c$, $\mathrm{P'F} = b - c$,

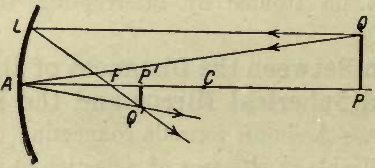

Fig. 66.

and $$\frac{a}{b} = \frac{c}{b - c},$$

or, $$ab - ac = bc.$$

That is, $$ac + bc = ab,$$

and dividing throughout by abc, we get—

$$\frac{1}{b} + \frac{1}{a} = \frac{1}{c}.$$

This is a numerical relation between a, b, and c, for this particular case of reflection at the surface of a concave mirror. This relation evidently applies also to the conjugate case in which the object is placed between F and C, and a real image is formed beyond C, at a distance from the mirror greater than its radius of curvature.

The case in which the object PQ is placed at a distance from the mirror less than its focal length, and the image P′Q′ is virtual, is represented in Fig. 67. The construction in this figure corresponds exactly with that of Fig. 66, and by the same steps as those given above we have—

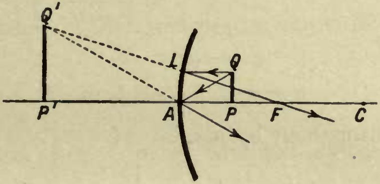

Fig. 67.

$$\frac{\text{AP}}{\text{AP}'} = \frac{\text{AF}}{\text{P}'\text{F}},$$

and as AP $= a$, AP′ $= b$, AF $= f$, and P′F $= b + c$, we have—

$$\frac{a}{b} = \frac{c}{b + c},$$

or $$ab + ac = bc.$$

That is, $$bc - ac = ab,$$

and, dividing throughout by abc, we get—

$$\frac{1}{a} - \frac{1}{b} = \frac{1}{c}.$$

This is the numerical relation between a, b, and c for the particular case just considered.

For reflection at the surface of a **convex mirror** there is only one general case (Art. 48) to be considered. This case is represented in Fig. 68, where P′Q′ is the virtual image of the object PQ. The construction of the figure corresponds with that of Figs. 66 and 67, and, in exactly the same way as before, we get—

$$\frac{AP}{A'P'} = \frac{AF}{P'F}.$$

In this case $AP = a$, $AP' = b$, $AF = C$, and $P'F = c -$ and, therefore,

$$\frac{a}{b} = \frac{c}{c - b},$$

or

$$ac - ab = bc.$$

That is,

$$ac - bc = ab,$$

and, dividing throughout by abc,

$$\frac{1}{b} - \frac{1}{a} = \frac{1}{c}.$$

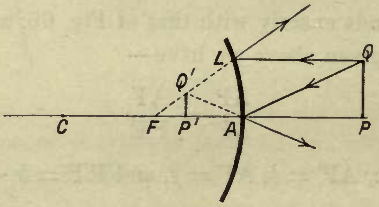

Fig. 68.

In this way we get a different numerical relation for each case considered. It is clear that it would be much more convenient to obtain a single relation that would apply to all cases of reflection at the surface of a spherical mirror. Such a relation can be obtained by adopting a convention by which distances measured in one direction along the principal axis of the mirror are considered positive, and distances measured in the opposite direction are considered negative. The usual sign convention is that by which distances measured along the axis in a direction *opposed* to the incident light are considered *positive*, and distances measured in the same direction as the incident light are considered negative.

Assuming this convention to hold, let u, v, and f denote the distances of the image, the object, and the focus from the pole of the mirror. That is, let u, v, and f denote, not merely the numbers measuring these distances, but *these numbers each with a positive or negative sign attached according to the above convention.*

Now, on referring to Fig. 66, it will be seen that the three distances AP, AP′, and AF, denoted respectively by u, v, and f, are all *positive* according to this convention. Hence, remembering that a, b, and c are the numerical measures of these distances, we have—

$$u = a,\ v = b,\ f = c,$$

and the relation $\frac{1}{b} + \frac{1}{a} = \frac{1}{c}$, obtained above from Fig. 66, becomes

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

It will be found that this formula is a general relation between the three quantities u, v, and f, and is applicable to all cases of reflection at the surface of a spherical mirror. The cases illustrated by Figs. 66, 67, and 68 differ essentially in the directions in which the lines AP, AP′, and AF are drawn from the point A, and the formula can, therefore, be made to apply to any particular case by giving the numerical values to be substituted for u, v, or f their proper signs according to the sign convention just explained.

Thus, in Fig. 67 we have—

$$u = a,\ v = -b,\ f = c,$$

and substituting these values for u, v, and f in the general relation we get the relation $\frac{1}{a} - \frac{1}{b} = \frac{1}{c}$, which is the same as that obtained geometrically from the figure.

Similarly, in Fig. 68 we have—

$$u = a,\ v = -b,\ f = -c,$$

and on substituting these values for u, v, and f in the general formula we get the relation $\frac{1}{b} - \frac{1}{a} = \frac{1}{c}$ already derived geometrically from the figure.*

The relation $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, associated with the sign convention given above, is thus a general relation applicable to all cases of reflection at the surface of a spherical mirror.

The form of the relation is necessarily the same as that of the numerical relation for the case in which all the distances involved are positive, according to the convention adopted (Fig. 66), and in proving the relation it is simplest to derive it directly from this case.

The relation may also be obtained from the geometrical result given in Art. 45.

Here, in Fig. 50, we have—

$$\frac{\mathrm{AP}}{\mathrm{AP}} = \frac{\mathrm{P'C}}{\mathrm{CP}},$$

and as AC = 2AF, and all the distances involved are positive in this figure, we may write—

$$\mathrm{AP} = u,\ \mathrm{AP}' = v,\ \mathrm{P'C} = 2f - v, \text{ and } \mathrm{CP} = u - 2f,$$

and, therefore,

$$\frac{v}{u} = \frac{2f - v}{u - 2f},$$

or

$$uv - 2fv = 2fu - uv.$$

---

* It is important to distinguish clearly between the u, v, f and the a, b, c relations given above. The formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ is an *algebraic* relation, in which the letters u, v, and f denote numbers *which may be positive or negative in sign*, according to a certain convention. The a, b, c formulæ are *numerical* or *arithmetical* relations in which the letters a, b, c denote merely ordinary numbers or arithmetical values *which are always positive in sign.*

That is, $$2fu + 2fv = 2uv,$$
or $$fu + fv = uv,$$
and, dividing throughout by *uvf*, we get—

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

In applying this relation, it must be remembered that all the distances are measured from the pole of the mirror, and that a positive or negative sign must be attached to the numerical measure of each distance, according to the sign convention explained above. It is well to note that by this convention the measure of the focal length is always positive for a concave mirror, and negative for a convex mirror.

Numerical Example.—An object is placed 30 cm. in front of a spherical mirror of 10 cm. focal length; find the position of the image when the mirror is (*a*) concave, (*b*) convex.

From the data of the question we have—

(*a*) $$u = +\,30,\ f = +\,10,$$

and $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

gives $$\frac{1}{v} + \frac{1}{30} = \frac{1}{10}.$$

That is, $$\frac{1}{v} = \frac{1}{10} - \frac{1}{30} = \frac{1}{15},$$

and, therefore, $v = +\,15$, or the image is formed 15 cm. *in front* of the mirror.

(*b*) $$u = +\,30, f = -\,10,$$

and $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

gives $$\frac{1}{v} + \frac{1}{30} = -\,\frac{1}{10}.$$

That is, $$\frac{1}{v} = -\,\frac{1}{10} - \frac{1}{30} = -\,\frac{2}{15},$$

and, therefore, $v = -7{\cdot}5$, or the image is formed 7·5 cm. *behind* the mirror.

It should be noticed from this example, and from others given in the text, that no sign is associated with the distance to be found; the calculation when finished will give, algebraically (as for v above), the numerical measure of the distance with its proper sign attached.

Experiment 32.—Set up an optical bench with a concave mirror, object, and screen. Focus a real image of the object on the screen for a number of different positions of the object, and measure for each adjustment the distances of the object and image from the mirror.

Calculate from the relation $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ the value of f for each pair of corresponding values of u and v, given by the experiment. The values of f should be practically constant, and their mean may be taken, with fair accuracy, as the focal length of the mirror.

This is another method of finding the focal length of a concave mirror.

The experimental data should also be obtained by means of a needle point, or paper arrow point, in daylight as in Exp. 28.

Example.—In an experiment of this kind when the distance of the object from the mirror was 30 cm., the distance of the real image was found to be 60 cm. Hence, in $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ we have $u = 30$, $v = 60$, and $\frac{1}{60} + \frac{1}{30} = \frac{1}{f}$. That is, $\frac{1}{f} = \frac{1}{20}$, or $f = 20$.

That is, the focal length of the mirror is 20 cm. The value of f is positive as it should be for a concave mirror.

51. **Relative Size of Image and Object.**—It has been explained in Art. 48 that the ratio of any linear dimension of the image to the corresponding linear dimension of the object is called the **magnification** of the image, and is usually denoted by m, or by writing the ratio $\frac{\text{image}}{\text{object}}$.

In all cases of reflection at the surface of a spherical mirror it can be seen from Figs. 66, 67, and 68 that

$$\frac{P'Q'}{PQ} = \frac{AP'}{AP}.$$

That is, $$\frac{\text{Image}}{\text{Object}} = \frac{\text{Distance of image from mirror}}{\text{Distance of object from mirror}}. \quad . \quad (1)$$

It can also be seen from figures having the construction shown in Fig. 69 that in all cases we have—

$$\frac{P'Q'}{AL} = \frac{P'F}{AF},$$

or, since $$AL = PQ,$$

$$\frac{P'Q'}{PQ} = \frac{P'F}{AF}.$$

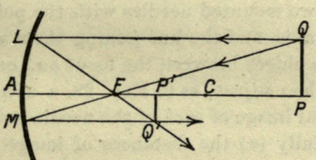

Fig. 69.

That is, $$\frac{\text{Image}}{\text{Object}} = \frac{\text{Distance of image from focus}}{\text{Focal length}}. \quad . \quad (2)$$

Also, in the same way,

$$\frac{AM}{PQ} = \frac{AF}{PF}$$

or, since $$AM = P'Q',$$

$$\frac{P'Q'}{PQ} = \frac{AF}{PF}.$$

That is, $$\frac{\text{Image}}{\text{Object}} = \frac{\text{Focal length}}{\text{Distance of object from focus}}. \quad . \quad (3)$$

Experiment 33.—Set up an optical bench with concave mirror, illuminated object, and screen as explained in Art 49. Place the object in front of the mirror at a distance from it greater than the radius of curvature. Focus the image of the object on the screen. Measure carefully (*a*) the distances of the object and image from the mirror; (*b*) the diameters of the circles on the object screen, and the diameters of the corresponding circles in the image; or, if narrow strip screens are used, the distances between corresponding lines on the object and image. From the measurements taken in (*b*) calculate the magnification of the image, and compare the result with the ratio of the respective distances of image and object from the mirror as given by the (*a*) measurements.

Repeat these measurements for different distances of object and image from the mirror. Make a similar set of measurements also with the object placed between the focus and the centre of curvature. It will be found that in all cases the results are in accordance with the relation given above.

Experiment 34.—Set up a concave mirror on a table with its axis horizontal, as explained in Exp. 28, and place in front of it a stand carrying two mounted needles with the points of the needles on a level with the axis, and the line joining them at right angles to the axis. Place this object between the focus and centre of curvature of the mirror and then adjust, as in Exp. 28, a needle point in coincidence with the real image of *each* of the needle points used as object.

Measure carefully (*a*) the distances of image and object from the mirror; (*b*) the distance between the two needle points of the object, and the corresponding distance between the images of these two points, as marked by the needles adjusted in coincidence with them.

From the measurement taken in (*b*) calculate the magnification of the image, and compare the result with the ratio of the distances of the image and object from the mirror.

Repeat these measurements for different distances of the object and image from the mirror, and also for the case where the object is placed beyond the centre of curvature. It will be found, as before, that the results are, in all cases, in accordance with the relation given above.

These measurements, unlike those of Exp. 33 which must be made in a darkened room, can be made in daylight.

A fourth relation can also be obtained between the size of the image and the size of the object. From any figure, such as Fig. 56, it can readily be seen that—

$$\frac{\text{Image}}{\text{Object}} = \frac{\text{Distance of image from centre of curvature}}{\text{Distance of object from centre of curvature}}.$$

This relation should be noted, but it is not so important as the three given above.

Experiment 35.—Set up an object before a concave mirror as in Exp. 33, or Exp. 34, and determine the magnification of the image, as described in these experiments. Then divide the distance on the principal axis, between the image and the object in the ratio given by the magnification, and mark the point of division on the axis. It will be found on testing, as in Exp. 26, that this point is the centre of curvature of the mirror.

Obviously in Fig. 66, the distance P′P is divided at C, so that $\frac{P'C}{CP}$ measures the magnification of the image.

It should be noted that in all these relations the magnification is written $\frac{\text{Image}}{\text{Object}}$, and the distance of the image from any point, when it occurs in the relation, appears as the first term (or image term) of the magnification ratio, and the distance of the object from any point, when it occurs, is always found as the second term (or object term) of the ratio.

These relations are best applied to any particular case by means of a figure representing the case. They are, however, readily expressed as arithmetical formulæ, in terms of *a*, *b*, and *c* as above, or as algebraic formulæ in terms of *u*, *v*, and *f*, and the sign convention. Thus, in terms of *a*, *b*, and *c*, we have—

$$m = \frac{a}{b}. \qquad . \quad . \quad . \quad . \qquad (1)$$

$$m = \frac{b \sim c}{c}. \qquad . \quad . \quad . \quad . \qquad (2)$$

$$m = \frac{c}{a \sim c}. \qquad . \quad . \quad . \quad . \qquad (3)$$

The formulæ, involving *u*, *v*, and *f* and the sign convention, are most simply obtained from Fig. 66, where all the distances concerned are positive, according to convention. Thus, we have—

$$m = \frac{v}{u}. \qquad (1)$$

$$m = \frac{v - f}{f}. \qquad (2)$$

$$m = \frac{f}{u - f}. \qquad (3)$$

In this form the formulæ are applicable to any case of reflection at the surface of a spherical mirror. It must be remembered, however, that u, v, and f here, as in Art. 50, denote the numbers measuring the distances of the object, the image, and the focus *from the pole of the mirror*, with a positive or negative sign attached, for direction, according to convention. It must be noted, too, that from these formulæ the value of m will always be negative when the object and image are on opposite sides of the mirror. Thus, in the formula $m = \frac{v}{u}$, the value of m must be negative, if the values of v and u are opposite in sign. Hence, for a real object, the value of m is always negative when the image is **virtual**.

Experiment 36.—Set up an optical bench with illuminated object screen, concave mirror, and image screen. The object screen should have small squares ruled on it, as explained in Art. 49.

Place the object in position so as to give a real image, and focus the image on the screen. Measure the distance between any two of the lines crossing the image focussed on the screen, and calculate the magnification of the image. Measure the distance of the object and image from the mirror, and

(1) Verify the relation $m = \frac{v}{u}$.

(2) Find the focal length of the mirror from the relation $m = \frac{v - f}{f}$.

(3) Find the focal length from the relation $m = \frac{f}{u - f}$.

(4) Find the focal length from the relation $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$.

It should be noted that if m is known, f can be calculated from a knowledge of either u or v.

Example.—In an example similar to that described above, the following measurements were made :—

An object placed at a distance of 50 cm. from the mirror gave a real image on the screen at a distance of 76 cm. from the mirror. The distances between certain lines on the object screen were known to be 10 mm., 15 mm., and 20 mm., and the distances between the corresponding lines on the image were found to be 15 mm., 23 mm., and 30·5 mm. respectively.

From these data we get—

$$m = \frac{15}{10} = 1{\cdot}50.$$

$$= \frac{23}{15} = 1{\cdot}53.$$

$$= \frac{30{\cdot}5}{20} = 1{\cdot}53.$$

The mean value of m is, therefore, given by $m = \frac{4{\cdot}56}{3} = 1{\cdot}52$, and, as the image is real, the sign attached to this value is positive.

Also, we have—

(1) $$m = \frac{v}{u} = \frac{76}{50} = 1{\cdot}52.$$

(2) $$m = \frac{v - f}{f}.$$

$\therefore$ $$1{\cdot}52 = \frac{76 - f}{f}.$$

That is, $1{\cdot}52 f = 76 - f$,

or, $2{\cdot}52 f = 76$,

and, therefore, $f = 30{\cdot}2$ nearly.

That is, the focal length of the mirror is 30·2 cm.

(3) $$m = \frac{f}{u - f}.$$

$\therefore$ $$1{\cdot}52 = \frac{f}{50 - f}.$$

That is, $76 - 1{\cdot}52 f = f$,

or, $2{\cdot}52 f = 76$,

and, therefore, $f = 30{\cdot}2$.

That is, the focal length of the mirror is 30·2 cm.

(4) $$\frac{1}{v}+\frac{1}{u}=\frac{1}{f},$$

$$\therefore \quad \frac{1}{76}+\frac{1}{50}=\frac{1}{f},$$

or, $$f=\frac{76\times 50}{126}=30{\cdot}2 \text{ nearly.}$$

That is, the focal length of the mirror is 30·2 cm. The sign attached to this value of f is positive, as it should be for a concave mirror.

If the distance of the object and image from the principal focus be denoted by x and x' respectively, then, from relations (2) and (3) above (p. 114), we get—

$$m=\frac{x'}{f}=\frac{f}{x},$$

and, therefore, $$\frac{x'}{f}=\frac{f}{x},$$

or, $$xx'=f^2.$$

This formula may be taken arithmetically or algebraically, involving the sign convention. If taken algebraically, x and x' denote the numbers measuring the distances of the object and image respectively *from the focus*, each with a positive or negative sign attached according to convention. Taken in this way, it is obvious that the values of x and x' must always be of the same sign, for f^2 is necessarily positive whether the value of f be positive or negative. That is, in any case of reflection at the surface of a spherical mirror, the object and image are on the same side of the principal focus of the mirror.

The relation $xx'=f^2$ is a convenient one to remember, but it is not very convenient to apply on account of the distances involved being measured from the focus and not from the pole of the mirror.

52. **Spherical Aberration.**—In dealing with reflection from spherical mirrors we have, in every case, considered the

mirrors to be of **small aperture**, and we have seen that parallel rays falling on such a mirror, in a direction parallel to the principal axis, are all reflected through one point, the principal focus of the mirror, midway between the centre of curvature and the pole of the mirror. We have also seen that, if the rays incident upon the mirror diverge from, or converge to, a common focus, the reflected rays also diverge from, or converge to, a common focus, conjugate to the focus of the incident rays.

It must be remembered, however, that these results are approximately true only when the aperture of the mirror is very small, and that they cease to be even approximately true when the aperture exceeds a comparatively low limit.

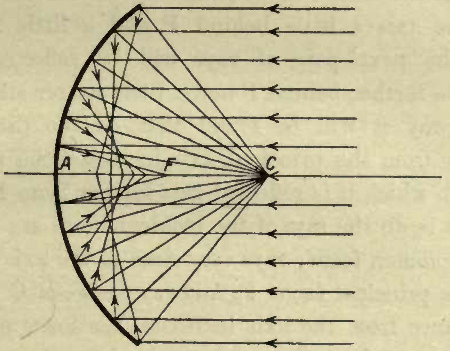

Fig. 70.

The truth of this statement will at once be realised if we draw a figure showing, as in Fig. 70, the reflection of a parallel beam of light incident on a concave mirror of large aperture in a direction parallel to the principal axis. The construction of the figure is given below, and the student will find it advantageous to make a careful drawing for himself on a large scale.

Draw a number of equidistant straight lines parallel to the principal axis to represent some of the rays of light in the parallel beam incident upon the mirror. From C, the centre of curvature, draw the radii, shown in the figure, to the points of incidence of these rays on the mirror; these radii are the normals to the mirror at the points of incidence. At each of these points set off the angle of reflection equal to the angle of incidence, and then draw lines to represent the reflected rays corresponding to the incident rays parallel to the principal axis. It will be found that the two incident rays nearest the principal axis are reflected through a point, F, on the axis almost exactly midway between C, the centre of curvature, and A, the pole of the mirror. The next two rays, a little further from the axis, will be found to be reflected through a point on the axis a little behind F and a little nearer to A; similarly, the next pair of rays will be reflected through a point a little further behind F and a little nearer still to A.

In this way it will be found that, as the distance of an incident ray from the principal axis increases, the point on the axis through which it is reflected gets further from F and nearer to A. That is, all the rays of the incident beam are not reflected through a common focus; rays very close to the axis are reflected through the principal focus F, midway between C and A; but, as the distance from the axis increases, the focus of all rays at the same distance from the axis travels along the principal axis from F towards A. If the aperture of the mirror is as great as 120° the focus of the outermost rays is exactly at A.

This change in the position of the focus of the reflected rays as the distance of the incident rays from the principal axis increases is due to the spherical form of the mirror, and is known as **spherical aberration.**

It will be clear from what has been said that the principal focus F of a spherical mirror, as defined in Art. 44, is the common focus of rays incident on the mirror, in a direction parallel to the principal axis, only when the aperture of the mirror is small.

In the same way it can be shown, by a method exactly similar to that adopted in Fig. 70, that when a divergent or convergent pencil is reflected at the surface of a spherical mirror the reflected rays have a common focus, at a single point conjugate to the focus of the incident pencil, only when the aperture of the mirror is small.

When the aperture of a spherical mirror, used to obtain the image of any object, is not small the pencil of rays incident on the mirror, from any point on the object, is not reflected through a *single point* in the image, but through a number of adjacent points; for this reason the image given by the mirror is badly defined, and the defect is said to be due to the spherical aberration of the mirror.

Experiment 37.—Take a concave mirror of large aperture, and place immediately in front of it a screen of paper with a hole in the centre, exposing only a small portion of the mirror round the pole. Place a convenient object in front of the mirror, and focus a real image of the object on a screen, taking care to get the image as sharply defined as possible. Now remove the screen in front of the mirror, so that reflection takes place over the whole surface of the mirror, and note the effect on the definition of the image.

It will be found that the image becomes blurred and indistinct, and that it cannot be sharply focussed by any adjustment of the position of the screen.

This lack of definition of the image formed by a mirror of large aperture is due to spherical aberration.

CHAPTER VI.

REFRACTION.

53. **Refraction.**—It has been explained, in Art. 10, that the path of a ray of light is a straight line so long as its course lies in the same homogeneous medium, but when it passes from one medium into another it, in general, undergoes a change of direction at the surface of separation of the two media.

This passage of a ray of light from one medium to another, accompanied, in general, by a change of direction at the surface of separation of the media, is called **refraction.**

Hence, when a ray of light in any medium is incident on the surface of separation of this medium from another, and suffers refraction at the point of incidence on the surface, the **refracted ray** is continued into the second medium in a direction differing from that of the incident ray. The angle which the incident ray makes with the normal to the surface of separation at the point of incidence, is called the **angle of incidence,** and the angle which the refracted ray makes with the normal is called the **angle of refraction.**

Experiment 38.—Get a rectangular tank, with plate glass sides and ends, about 24 inches long, 12 inches deep, and 9 inches wide. The bottom of the tank should be painted white on the inside. Fill it to a depth of about 10 inches with water, and put a few drops of fluorescene into the water, or make it milky with freshly precipitated silver chloride by adding small quantities of silver nitrate and hydrochloric acid, as in Exp. 13. Arrange a piece of plane mirror glass, M, as shown in Fig. 71, over the trough so that it can reflect a horizontal beam of light, such as *ab*, down into the water in the trough. The mirror should be supported, as in the clamp of a retort stand, so that it can be rotated round an axis at right angles to the plane of the paper in the figure. Arrange, also, a piece of mirror glass, N, in a horizontal position on the bottom of the tank. It is convenient for some purposes to substitute a shallow tray containing clean mercury for this mirror.

Now adjust an optical lantern to give a narrow but intense beam of parallel light, and, in a dark room, direct this beam, *ab*, in a plane parallel to the length of the trough on to the mirror M. Adjust the position of this mirror until the reflected ray *bc* is incident obliquely on the surface of the water at *c*, and the path of the reflected ray *cd* can be clearly traced in the milky water. Now adjust the position of the mirror N until this refracted ray is incident on it at *d*, and is reflected from it along *de* so as to be again incident on the surface of separation of the water and the air at *e*.

Fill the space over the tank with smoke, and note (*a*) the path of the incident ray *bc*, the refraction of this ray from air to water at *c*, and the path of the refracted ray *cd* in water; (*b*) the path of the ray *de*, the refraction of this ray from water to air at *e*, and the path of the refracted ray *ef* in air. Notice, also, that the ray *bc*, incident on the surface of separation of the air and the water at *c*, is partly reflected along *cd'* and partly refracted along *cd*, the refracted ray being more intense than the reflected ray. Similarly, the ray *de*, incident on the surface of separation at *e*, is partly reflected along *ef'* and partly refracted along *ef*; the path of the beam *ef'* can be clearly traced, and a bright patch can be seen at *f'* where it strikes the white bottom of the tank.

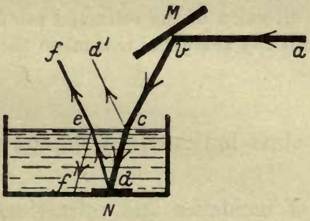

Fig. 71.

The following points should be carefully made out:—(1) At *c*, where refraction takes place from air to water, and at *e*, where refraction takes place from water to air, the angles of incidence and refraction are in the same plane—that is, at each point the incident ray, the refracted ray, and the normal at the point of incidence are in the same plane. In fact, if a tray of mercury take the place of the mirror at N, so that the normals at *c*, *d*, and *e* are parallel, it will be found that the rays *bc*, *cd*, *de*, and *ef*, together with the partially reflected rays *cd'* and *ef'* and the normals at *c*, *d*, and *e*, are all in the same plane. This can be clearly seen by looking along this plane from one end of the tank. (2) At *c*, where refraction takes place from *air to water*, the refracted ray is not continued in the direction of the incident ray, but *is bent* from this direction at the point of incidence *towards the normal.* (3) Similarly, at *e*, where refraction takes place from *water to air*, the refracted ray *is bent away from the normal.*

Change the position of the mirror M so as to change the angle of incidence of the ray *bc* on the surface of the water. It will be found that the same general results may be observed for any angle of incidence. As the angle of incidence at *c* and *e* increases it will, however, be noted (Art. 26) that the partially reflected beams *cd'* and *ef'* become stronger and the corresponding refracted rays *cd* and *ef* proportionately weaker.

Adjust the position of M so that the ray *bc* is directly incident on the surface of the water at right angles to the surface, and notice that in this case the refraction at *c* is not accompanied by change in direction; the direction of the refracted ray is simply a continuation of the incident ray along the normal to the surface at the point of incidence.

The important points indicated by the foregoing experiment are :—

(1) The angles of incidence and refraction are in the same plane.

(2) The angle of refraction is either less than or greater than the angle of incidence : when the ray passes from air to water the angle of refraction is less than the angle of incidence, and the refracted ray is bent towards the normal; similarly, when the ray passes from water to air, the angle of refraction is greater than the angle of incidence, and the refracted ray is bent away from the normal.

It can be shown by the wave theory of light that when a ray passes from one medium into another, in which the velocity of light is less than in the first, the refracted ray should be bent towards the normal, and when it passes into a medium in which the velocity of light is greater than in the first, the refracted ray should be bent away from the normal. As a general rule, however, the velocity of light is always less in the denser of any two media. Hence it may be said that when a ray of light passes from one medium into a *denser* one (as from air to water), the refracted ray is bent towards the normal, but when a ray passes from one medium into a *rarer* or *less dense* one (as from water to air), the refracted ray is bent away from the normal.

It can also be shown that the path of a ray of light refracted from one medium into another is reversible. Thus, in Fig. 71, the ray *bc* passing from air to water is refracted along *cd*, and the path *bcd* is reversible in the sense that a ray, *dc*, passing from water to air would be refracted along *cb*.

This is illustrated in the foregoing experiment by the refraction of the ray *de* from water to air along *ef*; the angle of incidence for the ray *de* at *e* is equal, by geometry, to the angle of refraction for the ray *cd* at *c*, and the angle of refraction for the ray *ef* at *e*, is shown by the experiment to be equal to the angle of incidence for the ray *bc* at *c*, so that the path *def* for light passing from water to air is practically a reversal of the path *bcd* for light passing from air to water.

This reversibility of the path of refraction from one medium to another, is merely a special case of the general principle of the reversibility of light. This principle states that if a ray of light can pass from a point *a* to a point *b* by any path, involving, it may be, both reflection and refraction, then a ray of light can pass from *b* to *a* by the same path reversed in direction.

54. **Sine of an Angle.**—Let AOB, Fig. 72, represent an acute angle. With centre O and radius O*a* describe the arc *ab*, cutting OA and OB in *a* and *b* respectively. From *b* draw *bc* perpendicular to O*a* and cutting it at *c*.

Now the ratio of the **arc** *ab* to the radius O*a* is constant for a given angle, whatever the length of the radius, and is proportional to the magnitude of the angle AOB; for example, if the angle is doubled or halved, the value of this ratio is also doubled or halved. Hence the value of this ratio may be taken as a measure of the angle; it is generally called the **circular measure** of the angle. The ratio of the **perpendicular** *bc* to the radius O*a* is also constant for a given angle, whatever the length of the radius, but it is not proportional to the angle in the sense explained above. The ratio is, however, definitely connected with the magnitude of the angle; it has a definite constant value for any given value of the angle, and it increases from 0 to 1 as the angle increases from 0° to 90°. This ratio of the perpendicular *bc* to the radius O*a* (or O*b*) is called the **sine** of the angle AOB.

In the right-angled triangle O*bc* the sine of the angle at O is evidently given by the ratio $\frac{bc}{Ob}$; the two other ratios between the sides of this triangle, $\frac{Oc}{Ob}$ and $\frac{bc}{Oc}$, are also connected with the magnitude of the angle AOB, and are known respectively as the **cosine** and the **tangent** of the angle.

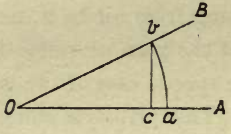

Fig. 72.

The sine of the angle AOB is usually written as sin AOB, or if the angle AOB be denoted by a, as sin a.

55. **Laws of Refraction.**—When a ray of light is refracted from one medium into another, the process of refraction is subject to two simple laws. The first law has already been noticed; it has been seen in Exp. 38 that the angle of refraction and the angle of incidence are always in the same plane—that is, the incident and refracted rays, and the normal at the point of incidence, are in one plane. The second law deals with the relation between the angles of incidence and refraction. It has

already been shown that these angles are not equal; it remains to determine between their magnitudes a definite quantitative relation which will apply generally to all cases of refraction.

Experiment 39.—A block of glass about 2 inches long, 1½ inches wide, and 1 inch thick, and three small knife edges of wood or metal, about 1 inch high, are wanted for this experiment. Pins may be used instead of the knife edges, but they are not so satisfactory in use.

Place the block of glass with one of its faces ABCD (Fig. 73) resting on a sheet of paper fixed on a drawing-board with drawing pins.

Place one knife edge, with an edge as at *a*, in contact with the face DC of the block, and another, with an edge as at *b*, in contact with the face AB of the block. Arrange that the line *ab* is inclined to the faces AB and CD, as shown in the figure.

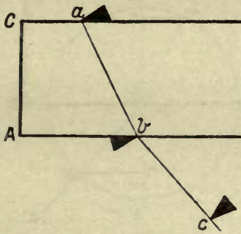

Fig. 73.

Now, take a third knife edge, and adjust its position in front of the block by "sighting" until the edge at *c* "covers" the edges at *a* and *b*, and the three edges appear to be exactly in the same plane.

This adjustment evidently serves to determine a path of refraction from glass to air; a ray, *ab*, in glass is incident on AB, the plane surface of separation of glass and air at *b*, and is refracted along *bc*, the direction determined in air by the edges at *b* and *c*.

It also indicates, by reversal, a path of refraction from air to glass; a ray, *cb*, in air is incident on the surface of the glass at *b*, and is refracted along *ba*, the direction determined in the glass by the edges at *a* and *b*. It can be verified, by sighting, that when the three edges at *a*, *b*, and *c* are in the same plane when seen from *c*, they are also in the same plane when seen from *a*.

Mark the points *a*, *b*, and *c* by a slight pressure on the knife edges before removing them. Draw the traces of the faces AB and

CD on the paper, then remove the glass block, and draw the lines *ab* and *bc* through the marked points at *a*, *b*, and *c*. Then, as shown in Fig. 74, draw the normal N*b*N at *b*, and, with centre *b* and any convenient radius, describe a circle cutting *ab* in *r* and *bc* in *s*. From the points *r* and *s* draw perpendiculars *rn* and *sn* to the normal N*b*N.

Measure very carefully the lengths of *rn* and *sn*, and calculate the value of the ratio $\frac{rn}{sn}$.

Now, repeat this experiment a number of times for different values of the angle *ab*N, and determine the value of the ratio $\frac{rn}{sn}$ from the data of each experiment.

It will be found that, although the actual values of *rn* and *sn* differ for different experiments, the value of the ratio $\frac{rn}{sn}$ is constant, and comes out, within the limits of experimental error, exactly the same for each determination.

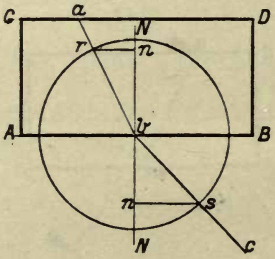

Fig. 74.

A number of adjustments can be marked on one diagram, if, in the experiment, the knife edge at *b* is kept fixed in position, and the edge at *c* adjusted for a number of different positions of the edge at *a*. Thus, in Fig. 75, four different positions of *a* and *c*, and the corresponding values of *rn* and *sn* are shown. The lengths of any two corresponding perpendiculars *rn* and *sn* can thus be measured, and the ratio $\frac{rn}{sn}$ calculated for each adjustment. It will be found that the value of the ratio is practically constant.

The result of this experiment, showing that the ratio of the perpendiculars *rn* and *sn* is constant for any two media, gives a definite quantitative relation between the angles of incidence and refraction when a ray of light is refracted from one of these two media into the other. These angles are evidently related in such a way that if a circle be drawn with the point of incidence on the surface of separation of the media as centre, and cutting the incident and refracted rays in the points r and s, then the ratio of the lengths of the perpendiculars from these points on to the normal at the point of incidence is constant for any angle of incidence and the corresponding angle of refraction.

This relation is, however, more concisely and conveniently expressed in terms of the **sines** of the angles of incidence and refraction. Thus, in Fig. 74, if ab be considered as the incident ray, and bc the refracted ray, then $\mathrm{N}ba$ is the angle of incidence, and $\mathrm{N}bc$ the angle of refraction. Also, $\frac{rn}{rb}$ is the sine of $\mathrm{N}ba$, and $\frac{sn}{sb}$ is the sine of $\mathrm{N}bc$, so that the ratio of the sine of the angle of incidence to the sine of the angle of refraction is $\frac{rn}{rb} : \frac{sn}{sb}$, or $\frac{rn}{sn}$, for rb is equal to sb. That is, the ratio of the lengths of the perpendiculars rn and sn is also the ratio of the sines of the angles of incidence and refraction.

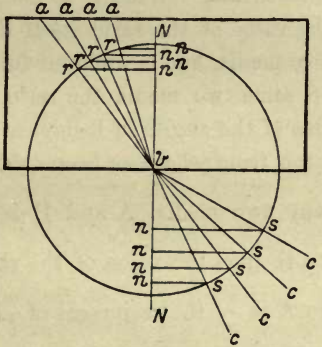

Fig. 75.

The quantitative relation between the angles of incidence and refraction, when light is refracted from one medium to another, may therefore be stated as follows :—

When light is refracted from one medium into another, then, for any two given media, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant for all values of these angles.

This relation between the angles of incidence and refraction is the second law of refraction, and is generally known as the **law of sines.**

It should be noticed that the essential part of this relation is that, for any two given media, the ratio of the sines of the angles of incidence and refraction *is constant for all values of these angles ;* the value of the ratio itself depends upon the properties of the two media, and is different for different pairs of media ; but for the same two media the ratio is constant for all corresponding values of the angles of incidence and refraction.

It will be clear, too, from what has been said, that if the value of the ratio for any two media, A and B is $\frac{a}{b}$ when light is refracted from A to B, then the value of the ratio when light is refracted from B to A is $\frac{b}{a}$, the reciprocal of $\frac{a}{b}$.

The laws of refraction may now be formulated in the following terms :—

1. *The angles of incidence and refraction lie in the same plane ; that is, the incident ray, the refracted ray, and the normal at the point of incidence, all lie in the same plane.*

2. *For the same two media, the ratio of the sines of the angles of incidence and refraction is constant for all corresponding values of these angles.*

56. **Index of Refraction.**—When a ray of light is refracted from one medium, A, into another, B, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant, whatever the values of these angles may be. The value of this constant ratio is called **the relative index of**

refraction from the medium A to the medium B, and is usually denoted by μ, or more precisely by ${}_{A}\mu_{B}$. If the relative index of refraction from medium A to medium B be denoted by μ, then the relative index of refraction from medium B to medium A is evidently denoted by $\frac{1}{\mu}$; that is ${}_{B}\mu_{A} = \frac{1}{{}_{A}\mu_{B}}$.

The relative index of refraction from a vacuum (that is from free ether) into any medium is generally called the **absolute index of refraction** for that medium.

The relative index of refraction from air into any medium is an important constant, and differs very slightly from the absolute refractive index of the medium. It is convenient to remember the following approximate values for glass and water :—

Refractive index, from air to glass, $= \frac{3}{2}$, from glass to air, $= \frac{2}{3}$.
Refraction index, from air to water, $= \frac{4}{3}$, from water to air, $= \frac{3}{4}$.

So far we have considered the index of refraction merely as a geometrical relation, established by experiment, between the directions of the incident and refracted rays. When considered in connection with the wave theory of light, a definite physical meaning can, however, be attached to this constant. It can be shown that the index of refraction from any medium A into another medium B is the ratio of the velocity of light in A to its velocity in B.

The absolute index of refraction for any medium is, therefore, the ratio of the velocity of light in free ether to the velocity of light in the medium. This ratio differs for light waves of different wave length, being greater the shorter the wave length, and, as difference of wave length in light waves corresponds to difference in colour, it follows that the value of the absolute refractive index for any medium depends upon the colour of the light which suffers refraction. The light having the greatest wave length and lowest refractive index is of a deep red colour, and that of the shortest wave length and highest refractive index is of a violet colour; between these two extremes the refractive index increases as the wave length decreases, and the colour of the light shades off from red through orange, yellow, green, and blue, to violet.*

Table of Refractive Indices.

(Mean Values.)

Substance	Index	Substance	Index
Diamond	2·60	Oil of turpentine	1·47
Iceland spar	1·65	Alcohol	1·37
Flint glass	1·62	Ether	1·36
Rock-crystal	1·55	Water	1·33
Crown glass	1·53	Carbonic acid gas	1·00045
Ice	1·31	Air	1·00029
Carbon disulphide	1·63	Hydrogen	1·00014

The refractive index given in this table for any substance is the mean of its two extreme values for red and violet light respectively.

The relative index of refraction for any two media can be determined experimentally by the method of Exp. 39; the ratio $\frac{rn}{sn}$ (Fig. 74) is evidently the index of refraction from glass to air, and $\frac{sn}{rn}$ the index of refraction from air to glass. Exps. 41 and 43 may also be employed for this purpose.

57. **General Construction for Determining the Path of the Refracted Ray.**—When a ray of light is refracted from one medium into another, and the relative index of refraction for the two media is known, the refracted ray corresponding to any given incident ray can be determined by a simple geometrical construction.

Let *ab*, Fig. 76, represent a ray of light travelling from the medium A into the medium B, and incident on the plane surface of separation of the two media at the point *b*. Let *de* be the trace of the surface of separation of the media, and *nbn* the normal to the surface at *b*, the point of incidence.

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* It will be understood from this, that this chapter and the next deals with the refraction of monochromatic light, or homogeneous light of a particular wave length.

If μ denote the index of refraction from A to B mark off from *b* along *be* and *bd*, respectively, the distances *bg* and *bf*, such that the ratio $\frac{bg}{bf} = \mu$. That is, if *bg*, taken on the same side of the normal as the incident ray, is μ units long, *bf* should be one unit long; or if μ is known as a vulgar fraction, $\frac{a}{b}$, then *bg* and *bf* should be *a* and *b* units long respectively.

From *g* draw *gr* at right angles to *be* and cutting *ab* at *r*; then with *b* as centre and *br* as radius describe a circle, and from *f* draw *fs* at right angles to *bd*, to cut the circumference of this circle at *s*. Then the line *bc*, drawn from *b* through this point *s*, is the direction of the refracted ray. For, if *rn* and *sn* be drawn from *r* and *s*, respectively, perpendicular to the normal *nbn*, we evidently have—

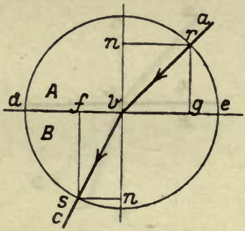

Fig. 76.

$$\frac{rn}{sn} = \frac{bg}{bf} = \mu,$$

that is, *bc* is the refracted ray corresponding to the incident ray *ab*.

In Fig. 76 the surface of separation of the media is a plane surface; the construction here given is, however, applicable to a surface of any form if *dbe* be taken to represent the trace of the plane tangential to the surface at the point *b*, and *nbn* the normal to the surface at this point.

58. **Deviation Caused by Refraction.**—When a ray of light is refracted from one medium into another, it, in general, undergoes a change of direction or suffers **deviation** from its original course. Thus, in Fig. 77, the ray *ab*, incident on the surface of separation of the media A and B at *b*, is refracted along *bc*, and the change of direction or **deviation** which accompanies the refraction of the ray is evidently measured by the angle *b′bc* which lies between *bb′*, the direction of the incident ray, and *bc*, the direction of the refracted ray.

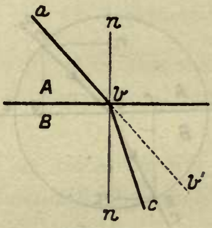

Fig. 77.

The angle *b′bc* is evidently the difference between the angle *nbb′* and *nbc*. But *nbb′* is equal to *nba*, the angle of incidence, and *nbc* is the angle of refraction; the deviation of the ray is, therefore, equal to the difference between the angles of incidence and refraction. That is, if i denote the angle of incidence and r the angle of refraction, then D, the deviation, is given by $D = i - r$.

If *cb* be taken as the incident ray and *ba* as the refracted ray, then the angle *b′bc* still measures the deviation of the ray and $D = r - i$, for, in this case, *nbc* is the angle of incidence and *nba* the angle of refraction. In either case the deviation is evidently equal to the difference between the angles of incidence and refraction.

Experiment 40.—Set up the apparatus of Exp. 39 and find out by direct measurement whether the deviation of a ray is constant, whatever the magnitude of the angle of incidence may be; if it is not constant, find how it changes as the angle of incidence increases.

It will be found that the deviation increases as the angles of incidence and refraction increase, thus proving that the greater of these two angles increases more rapidly than the other. That is, if *i*, the angle of incidence, is greater than *r*, the angle of refraction, then equal increments in *r* correspond to gradually increasing increments in *i*, or equal increments in *i* mean gradually decreasing increments in *r*. This may also be verified graphically by drawing a few figures showing the deviation a ray suffers in passing from one medium to another at different angles of incidence.

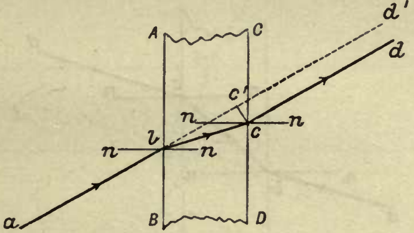

Fig. 78.

59. **Refraction of a Ray through a Plate of any Medium.**

—A plate of any medium is a portion of the medium contained between two parallel plane faces separating it from the surrounding medium. Let ABCD, Fig. 78, represent a portion of a plate of a medium denser than the surrounding medium—for example, a plate of glass in air. The faces at AB and CD are plane parallel faces at right angles to the plane of the paper. Let *a b* represent a ray of light incident on the face AB at the point *b*. This ray is refracted at *b* into the plate, and if the index of refraction from the external medium into the medium of the plate be known, the path of the refracted ray can be determined by the construction given in Art. 57. In this case, since the medium of the plate is denser than the surrounding medium the refracted ray is bent towards the normal at *b*. Let *bc* be the direction of the refracted ray within the plate, and let it be incident on the second face CD of the plate at *c*; the ray is now refracted at *c* into the external medium, and the path of the emergent refracted ray *cd* can be determined exactly, as at *b*, by the construction of Art. 57.

Similarly, when the medium of the plate is rarer or less dense than the surrounding medium, the general path of the ray *abcd* through the plate is as shown in Fig. 79, and the exact path of the ray for the given media can be determined by applying the construction given in Art. 57 at the points *b* and *c*, as already explained.

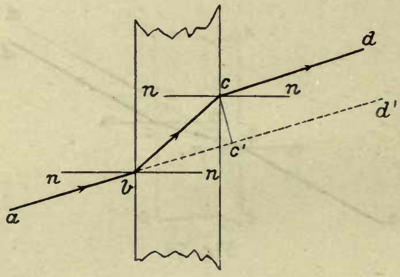

Fig. 79.

It will be seen from the geometry of the figures that the emergent ray *cd* is parallel to the entering ray *ab*. For since the faces AB and CD of the **plate** are parallel (by definition) the normals at *b* and *c* must be parallel; the alternate angles *nbc* and *ncb* are therefore equal, and the angles *abn* and *dcn*, being similarly related by the law of sines to these equal angles, must themselves be equal. The rays *ab* and *cd* thus make equal angles with the **parallel** normals *nbn* and *ncn*, and are therefore parallel to each other.

Hence it will be seen that a ray of light suffers no change of direction or deviation by refraction through a plate of any medium. It undergoes, however, a small *lateral displacement* from its original path; thus the emergent ray *cd* is parallel to *abc′d′*, the direction of the incident ray, but it is displaced laterally from this path by the distance *cc′*. The magnitude of this lateral displacement increases with the angle of incidence, and is evidently towards the normal at *b* or away from this normal, according as the medium of the plate is denser or rarer than the surrounding medium. When the ray is **directly** or **normally** incident on the plate it passes straight through without suffering deviation or displacement of any kind.

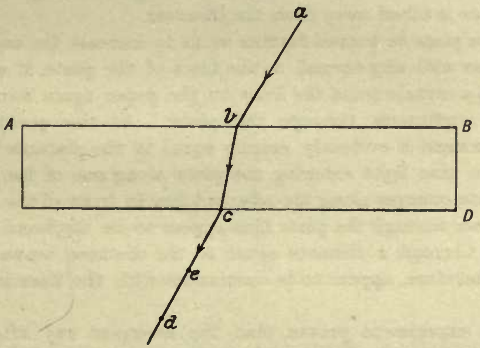

Fig. 80.

Experiment 41.—Fix a sheet of plain unruled paper on a drawing board, draw a short straight line on the paper, and then place a rectangular plate of glass standing on the paper with its parallel faces vertical, and in such a position that the line is inclined to one of these faces at an angle of about 45°.

In Fig. 80, let ABCD represent the trace of the plate on the paper, and *ab* the straight line.

Now look through the plate, from the side CD, and make two dots *d* and *e* on the paper, so that they appear exactly in a line with *ab* as seen through the plate. Mark the traces of the sides AB and CD with a sharp pencil, and then remove the glass plate. Draw a line through the dots *d*, *e*, cutting the trace of CD in *c* and then join *bc*. The path *abced* is evidently the course of a ray of light, incident along *ab*, through the plate.

Verify that the emergent ray *ced* is parallel to the incident ray *ab*. Also measure the lateral displacement of *ced* for a number of different angles of incidence at *b*, and plot a curve showing how the displacement increases with the angle of incidence.

Experiment 42.—Place a plate of glass, as in Exp. 41, standing on a sheet of ruled foolscap with the parallel faces at right angles to the lines. Look along the lines through the plate; if the plate is exactly at right angles to the lines they will appear quite straight and continuous through the plate. Now turn the plate so that the lines make a small angle with a normal to the parallel faces; the lines seen through the plate will still appear to be parallel to their continuations in front of the plate, but they will be displaced slightly to the right or left of the observer, according as the right or left end of the plate is tilted away from the observer.

If the plate be turned further so as to increase the angle made by the lines with any normal to the faces of the plate, it will be found that at a certain point the lines on the paper again seem to be perfectly continuous through the plate. At this point the lateral displacement is evidently exactly equal to the distance between the lines, so that light entering the plate along one of the lines behind the plate emerges along the adjacent line in front of the plate. The lines seen through the plate thus appear to be displaced to the right or left through a distance equal to the distance between the lines, and, therefore, appear to be continuous with the lines in front of the plate.

This experiment proves that the emergent ray after refraction through a plate is parallel to the incident ray. If this were not so, the apparent direction of the lines seen through the plate would not be parallel to the lines in front of the plate.

Experiment 43.—Arrange for an experiment similar to Exp. 41. Draw two parallel straight lines on the paper, at a distance apart equal to about two-thirds of the thickness of the glass plate or block. It is convenient to draw the lines in the relative position shown in Fig. 81, at *ax* and *dy*. Place the plate of glass ABCD on the paper, so as to cover the ends *x* and *y* of the lines *ax* and *dy*. Then turn the plate as shown in the figure, until on looking along the line *dy* this line appears to be exactly continuous with *xa* as seen through the plate. Mark the traces AB and CD on the paper, by lines cutting the lines *ax* and *dy* at *b* and *c* respectively. Remove the plate and join *bc*. The path *abcd* is evidently a path of refraction through the plate; light entering the plate along *ab*, from any point on this line is refracted along *bc*, and emerges along *cd* in a direction parallel to *ab*, so that to an eye looking along *dc*, the path *dcba* appears as a straght continuous line.

Now determine as in Exp. 39, the value of the ratio $\frac{rn}{sn}$ for the refraction from air to glass at *b*, and also for the refraction from glass to air at *c*. The value of the ratio gives, in the former case, the relative index of refraction from air to glass, and, in the latter case, the relative index of refraction from glass to air. The one value should be the reciprocal of the other.

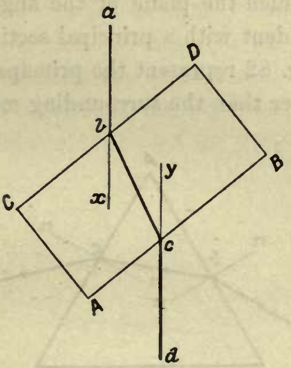

Fig. 81.

60. Refraction of a Ray through a Prism of any Medium.

—From an optical point of view a **prism** of any medium is a portion of the medium lying between two plane faces which are not parallel, as in the case of a plate, but inclined to each other at any angle. The line of intersection of these faces is known as the **edge** of the prism, and a section of the prism at any point in its length at right angles to this edge is called a **principal section.** The **refracting angle** of the prism is the angle between its faces as measured by the corresponding plane angle of the principal section.

The prisms generally used for optical experiments are triangular prisms in the geometrical sense of the term. The principal sections of such prisms may be equilateral, isosceles, or scalene triangles, according to the purpose for which the prism is intended. When the section is equilateral the angle at each edge is 60°, and thus there is no gain in having three edges to the prism; with an isosceles section there are two different angles available, and with a scalene section the angle at each edge is different, and thus the prism is equivalent to three prisms, considered in the optical sense.

In dealing with refraction through a prism, we shall consider only the case in which the plane of the angles of incidence and refraction is coincident with a principal section of the prism.

Let ABC in Fig. 82 represent the principal section of a prism of a material denser than the surrounding medium, and let BAC

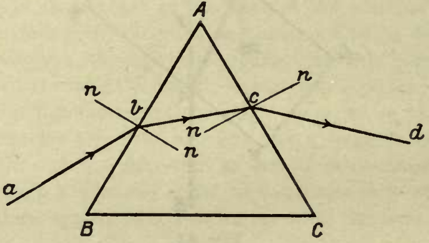

Fig. 82.

be the **refracting angle** considered. This implies that the faces, of which AB and AC are the traces, and which are inclined at the angle BAC, are to be taken as the **faces** of the prism, and that A is the **edge** of the prism.

Let *ab* represent a ray of light incident on the face AB of the prism at *b*. This ray is refracted at *b* into the prism, and if the index of refraction from the external medium into the medium of the prism be known, the exact path of the refracted ray can be determined by the construction given in Art. 57. In this case, since the medium of the prism is supposed to be denser than the surrounding medium, the refracted ray is bent towards the normal at *b*. Let *bc* be the direction of the refracted ray within the prism, and let the ray be incident on the second face, AC, of the prism at *c*; the ray is now refracted at *c* into the external medium, and the path of the emergent refracted ray *cd* can be determined exactly as at *b* by the construction of Art. 57.

Similarly, when the medium of the prism is rarer, or less dense than the surrounding medium, the general path of the ray *abcd* through the prism is as shown in Fig. 83, and the exact path of the ray can be determined by applying the usual construction at the points *b* and *c*, as already explained.

It will be seen from these figures that the emergent ray *cd* is not parallel to the entering ray *ab*, and that the refraction of a ray of light through a prism is therefore accompanied by deviation measured by the angle between the directions of *ab* and *cd*.

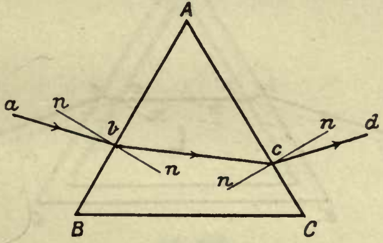

Fig. 83.

When the material of the prism is denser than the surrounding medium, the deviation, as shown in Fig. 82, is *away from the edge of the prism,* but when the material is less dense than the surrounding medium, the deviation, as shown in Fig. 83, is *towards* the edge of the prism. In practice prisms are generally used in air, and made of material denser than air, so that the case represented by Fig. 82, where the deviation is away from the edge of the prism, is the only one of practical importance.

Prisms of liquid media are sometimes used. The liquid is placed in a hollow prism, usually made of glass, and so constructed that the sides adjacent to the faces of the liquid prism are thin plates with truly parallel faces. When a ray is refracted through these sides and the liquid prism between them, the deviation produced is due entirely to the action of the prism, for, as explained in Art. 59, the thin plates which form the sides produce no deviation.

Fig. 84 shows the path of a ray, *abb′c′cd*, refracted through a hollow-glass prism filled with a liquid, **L**. At *b* the ray is refracted from air to glass, and at *b′* from glass to the liquid; then at *c′* from the liquid to glass, and finally, at *c* from glass to air. The dotted lines *a′b′* and *c′d′* show the paths in air corresponding to the path *b′c′* in the prism, assuming the glass sides of the hollow prism to be removed; these lines are parallel to *ab* and *cd* respectively, and obviously the deviation of the ray is the same when refracted through the prism of liquid and the glass sides of the hollow prism containing it, as if it were refracted directly through the liquid prism.

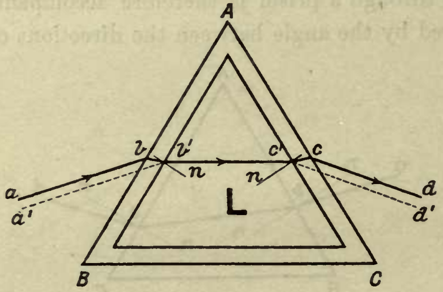

Fig. 84.

Experiment 44.—Arrange the apparatus of Exp. 41 for a similar experiment with a glass prism instead of a glass plate. The faces of the prism should be about an inch wide, and the ends of the prism should be at right angles to the length of the prism. Place the prism standing on one end with one of its faces, AB (Fig. 85), inclined to the line drawn on the paper, as shown in the figure. Then look through the prism from a point in front of the face AC, and make two dots *d* and *e* on the paper, exactly in a line with the direction of *ab* as seen through the prism. Draw the traces of the faces AB and AC on the paper, then draw a line through the dots *d* and *e*, cutting the trace of AC at *c*, and join *c* to the point *b* where the trace of AB cuts the line *ab*. The path *abcd* thus found is a path of refraction through the prism; light entering the prism along *ab* is refracted at *b* along *bc*, and emerges along *cde*, so that to an eye looking along *edc* the whole path *edcba* appears as a straight line.

Determine the index of refraction from air to the glass of the prism by applying the method of Exp. 39 to the refractions at *b* and *c*.

Measure also the deviation for a number of different paths through the prism, and plot a curve showing how the deviation varies with the angle of incidence at *b*. Construct the path of minimum deviation from this curve.

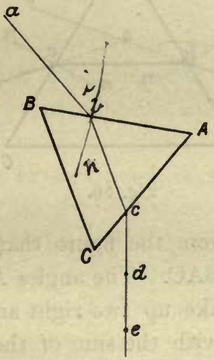

Fig. 85.

61. The Deviation of a Ray Refracted through a Prism.

—Let Fig. 86 represent the refraction of a ray, *abcd*, through the prism ABC in the case where the material of the prism is denser than the surrounding medium.

If i denote the angle of incidence *abn*, and r the angle of refraction *nbc*, then, by the result obtained in Art. 58, the deviation due to the refraction of the ray at the point *b* is given by $(i - r)$. Similarly, if i' and r' denote the correspondingly *placed* angles *ncd* and *bcn*, the deviation due to the refraction at the point *c* is given by $(i' - r')$. The total deviation of the ray caused by its refraction through the prism is, therefore, given by $D = (i + i') - (r + r')$.

This result can also be obtained directly from the figure; for the total deviation D is represented by the angle *b′od*, and by *Euc.* i. 32, we have—

$$\angle b'od = \angle obc + \angle bco.$$

That is, $$\mathrm{D} = (i - r) + (i' - r'),$$

or $$\mathrm{D} = (i + i') - (r + r').$$

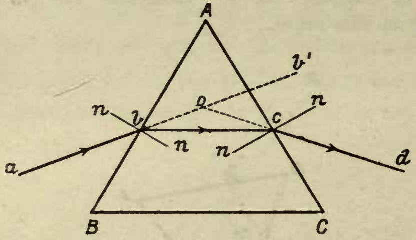

Fig. 86.

It can also be seen from the figure that $(r + r')$ is equal to the angle of the prism BAC. The angles A*bc* and A*cb*, together with the angle BAC, make up two right angles; and the angles A*bc* and A*cb*, together with the sum of the angles *nbc* and *bcn*, also make up two right angles. It follows, therefore, that—

$$\angle \mathrm{ABC} = \angle nbc + \angle bcn.$$

That is, if the angle ABC be denoted by A, we have—

$$\mathrm{A} = r + r'.$$

The total deviation of the ray is, therefore, given by—

$$\mathrm{D} = (i + i') - \mathrm{A}.$$

This result shows that unless $(i + i')$ be constant the deviation must vary with the value of i; it can be shown theoretically that the value of $(i + i')$ cannot be constant, and it is found experimentally that the deviation varies with i in such a way as to pass through a *minimum value* for a particular path of the ray through the prism.

If the ray passes symmetrically through the prism, as in Fig. 86, so that $i = i'$ and $r = r'$, the deviation of the ray is given by

$$D = 2i - 2r = 2i - A,$$

and it can be shown that for this path of the ray the deviation is a minimum.

The deviation for any path of refraction through the prism is given, as we have just found, by

$$D = (i + i') - (r + r') = (i + i') - A.$$

Any change in the path involves a change in each of the four angles i, i', r, and r', and the changes in i and i', as well as in r and r', are necessarily of opposite sign, for when one angle increases the other decreases. Since $r + r' = A$, and is, therefore, constant, the algebraic sum of the changes in r and r' is zero. The change in the deviation D, which attends any change in the path of the ray through the prism, depends, therefore, upon the changes in i and i', and is equal to the algebraic sum of the changes in these angles.

Now, when the ray passes symmetrically through the prism, so that $i = i'$ and $r = r'$, any change in the path of the ray means equal changes of opposite sign *in the same value of* r, at the points b and c in the path, and the corresponding changes in i and i' are, therefore, unequal (Art. 58), the increment of the one which increases being greater than the decrement of the one which decreases. The algebraic sum of the changes in i and i' is, therefore, positive for any change from this path; that is, any change from this path is accompanied by an *increase* in the deviation, and the path is, therefore, *the path of minimum deviation.* The minimum deviation for a ray refracted through a prism is, therefore, given by

$$D = 2i - A,$$

where i denotes the angle of incidence of the ray at the first face of the prism and also the angle of emergence of the ray at the second face of the prism.

From this relation we get—

$$2i = A + D,$$

or

$$i = \frac{A + D}{2}.$$

We also have—

$$A = 2r, \text{ or } r = \frac{A}{2},$$

where r denotes the angle of refraction corresponding to the angle of incidence i.

Now, μ the index of refraction of the material of the prism is given by

$$\mu = \frac{\sin i}{\sin r}.$$

That is,

$$\mu = \frac{\sin \frac{A + D}{2}}{\sin \frac{A}{2}}.$$

This relation is of great practical importance, for it is the basis of the most accurate experimental method of determining the refractive index of a transparent material.

The material is cut in the form of a prism, or, if liquid, it is enclosed in a hollow prism (Art. 60); the angle A of the prism and the minimum deviation, D, for a ray refracted through the prism, are then carefully measured by specially accurate optical methods, and the value of μ is deduced from the relation just given.

The following simple experiment illustrates the principle of this method of determining μ, but the apparatus required to make an accurate determination is of a very different character.

Experiment 45.—Take a sheet of stout cardboard and cut out a small disc from the centre of the sheet by means of a large cork borer.

Preserve the disc for future use, and pin the sheet of cardboard down flat on a drawing board. Get a glass prism, as in Exp. 44, and fix the disc with seccotine, or some such material, to the bottom of the prism, with its centre on the bisector of the angle of the prism and as near to the edge as possible, without allowing the disc to overlap the section of the prism. When the disc is securely fixed, place the prism on the sheet of cardboard so that the disc fits into the hole out of which it was cut, and, if necessary, file the edges of the disc so that it will rotate smoothly and easily in the hole.

Now arrange for an experiment similar to Exp. 44. Draw the line *ab* (Fig. 85) on the cardboard, in a direction passing through the centre of the disc on which the prism is fixed, and then rotate the prism till the face AB comes into the position relative to *ab*, shown in Fig. 85.

Then look through the prism from a point in front of the face AC, and note how the apparent direction of the line *ab* changes as the prism is rotated in either direction.

It will be found that the angle which this apparent direction of *ab* makes with its real direction (that is, the deviation for the path *abcde* through the prism) can generally be decreased by rotating the prism in a particular direction, but that, in all cases, when the prism comes into a certain position, this angle ceases to decrease and begins to increase, although the rotation may still be continued in the same direction as at first. That is, in looking through the prism from a point *e* in front of the face AC, the apparent direction of *ba* changes, so that, as the apparent position of the point *a* moves to the left of the observer (showing a *decrease* in the angle of deviation), and the rate of decrease becomes slower and slower until, finally, for a certain position of the prism, the apparent position of the point *a* stops moving to the left, and begins to move to the right without any change being made in the direction of rotation of the prism.

This position of the prism at which the angle of deviation ceases to decrease, is evidently the position of minimum deviation for a ray of light entering the prism at *b* along the line *ab*.

Set the prism in this position and mark the path of emergence *cde* by means of the dots *d* and *e*, as explained in Exp. 44.

Then measure carefully the angle between the directions of *ab* and *cde*, and also the angle BAC of the prism. These measurements give the angles D and A respectively of the formula,

$$\mu = \frac{\sin \dfrac{A + D}{2}}{\sin \dfrac{A}{2}},$$

and the value of μ can, therefore, be calculated from the data of the experiment.

The values of sin $\frac{A + D}{2}$ and sin $\frac{A}{2}$ can always be obtained graphically by drawing the angles with a protractor, on a fairly large scale, and then working out by measurement and calculation the value of the sine ratio, as defined in Art. 54.

This form of the experiment with a single ray is a little troublesome in practice, because the emergent ray is subject to displacement as well as change of direction. The displacement, however, is negligible when the prism is near the position of minimum deviation, so that it is quite possible to determine this position with fair accuracy by the method described.

A more satisfactory form of the experiment is described below.

Attach the disc of cardboard to the bottom of the prism, with its centre not on the medium line as described above, but on the edge AC, at the point *c*. Now place the prism in position on the cardboard sheet, and mark the lines AB and AC lightly, with pencil, on the sheet. Remove the prism and draw, in a direction parallel to that indicated generally by *ab* in Fig. 85, a number of parallel lines very close together, to represent a wide pencil of parallel rays. The width of the pencil should be equal to the distance *c*A on the A side of the ray which would, if produced, pass through *c*, and equal to *c*B on the B side of this ray. Replace the prism, mark the point *c* (the centre of the disc), clearly with a pencil dot and then, with *c* as centre and a radius of about 3 inches, describe an arc on which the different positions of the point *e*, for different directions of the ray emerging from the prism at *c*, can be marked. The point *d* is not wanted in this experiment; the dot at *c* is intended to take its place.

Now, set the prism in any position not too near the position of minimum deviation for the parallel pencil represented by the lines drawn on the cardboard. Look through the prism from a point in front of the face AC, and mark, on the arc just drawn, the position of a point *e*, which is so placed that the direction *ec* is exactly in a line with, or parallel to any one of the pencil of parallel lines seen through the prism. Now rotate the prism so as to reduce the deviation and follow the position of the point *e* along the arc. It will be found that, as the prism is rotated, the point *e* moves to the right (as the apparatus is arranged in Fig. 85) along the arc up to a certain point and then turns back and moves to the left. This shows that during the rotation of the prism the deviation decreases to a minimum and then increases.

Mark the direction of the emergent ray *ce* for the position of minimum deviation, and then measure D, the angle of minimum deviation. The value of μ can then be calculated from the values of D and A, as explained above.

It should be noticed that, in this form of the experiment, the emergent ray fixed by the dots at *c* and *e* corresponds, in different positions of the prism, to different incident rays of the parallel pencil incident on the face AB; if the pencil is wide enough, some one of the rays incident on AB will, in any position of the prism, emerge after refraction through the prism at the point *c*. Unless the parallel lines representing the pencil are drawn very closely together, this particular incident ray may not be represented by a line, but even if it lie between two lines, its direction is fixed with sufficient accuracy by the direction of the two parallel lines between which it lies.

It should be noted that, for a prism of given material, the deviation produced by **a prism of small angle** is approximately proportional to the angle. Thus, since

$$\mu = \frac{\sin \frac{A + D}{2}}{\sin \frac{A}{2}}$$

we have—

$$\mu = \frac{\frac{A + D}{2}}{\frac{A}{2}} = \frac{A + D}{A},$$

if A is small, for, if A is small, D is also small, and *for small angles* the ratio of the angles is approximately equal to the ratio of their sines.

Hence, we have

$$\mu = \frac{A + D}{A},$$

or

$$D = (\mu - 1) A.$$

That is, for a given value of μ, D is directly proportional to A.

For crown glass $(\mu - 1)$ is approximately equal to $\frac{1}{2}$, and, therefore, for a glass prism of small angle D is roughly equal to $\frac{A}{2}$.

62. Refraction of a Small Pencil of Rays Directly Incident on the Plane Surface of Separation of two Media.—A pencil of rays is said to be directly incident on a surface when the axis of the pencil is normal to, or at right angles to the surface. When a small pencil is directly incident on the surface of separation of two media, and is refracted at the surface from one medium to the other, the axial ray of the pencil being normal to the surface, passes on without deviation, thus becoming the axis of the refracted pencil, and the focus of the refracted pencil is at a point on the normal which forms the common axis of the incident and refracted pencils.

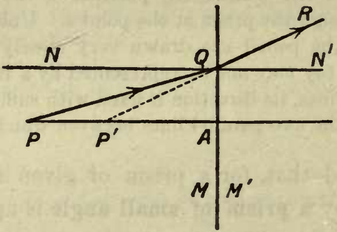

Fig. 87.

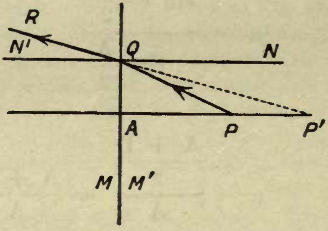

Fig. 88.

Thus in Fig. 87 and Fig. 88 let PA represent the axis of a small pencil of rays diverging from P, and incident directly along the normal PA, on the surface of separation of two media M and M′, of which M is the denser medium. Also, let PQ represent any one ray of this pencil incident on the surface of separation at Q. The axial ray PA is refracted from one medium to the other without deviation, and the ray PQ is refracted along QR in a direction such that if NQN′ be the normal at Q $\frac{\sin PQN}{\sin RQN'} = \mu$, where μ is the index of refraction for the direction in which the pencil is travelling.

Produce QR backwards to cut PA at P′. Then from the figure we have—

$$\angle PQN = \angle APQ,$$

and

$$\angle RQN' = \angle AP'Q,$$

and, therefore,

$$\frac{\sin APQ}{\sin AP'Q} = \mu.$$

But

$$\sin APQ = \frac{AQ}{QP},$$

and

$$\sin AP'Q = \frac{AQ}{QP'},$$

$$\therefore \qquad \frac{QP'}{QP} = \mu.$$

But if AQ is small—that is, if the pencil is of small angle—then AP′ and AP are approximately equal to QP′ and QP respectively, and, therefore,

$$\frac{AP'}{AP} = \mu,$$

or,

$$AP' = \mu\, AP.$$

That is, for two given media, the position of P′ is fixed by the position of P, *and is the same for all rays of a small pencil.* That is, P′ is the focus of the refracted pencil, and its distance from the surface of separation of the media is μ times the distance of P, the focus of the incident pencil, from the surface.

Hence, if the focus of the incident pencil be called the **object**, and the focus of the refracted pencil the **image**, it may be said that when refraction takes place along a normal to the surface of separation of two media, both image and object lie on the normal, and the distance of the image from the surface is μ times the distance of the object from the surface, where μ is the refractive index for the direction in which refraction takes place.

When the pencil is refracted from the denser to the less dense of the two media, as in Fig. 87, μ is less than 1, and AP′ is less than AP. When, however, the pencil is refracted from the less dense to the denser of the two media, as in Fig. 88, μ is greater than 1, and AP′ is greater than AP. In either case, AP′ = μ AP, provided μ denotes the refractive index for the direction in which refraction actually takes place.

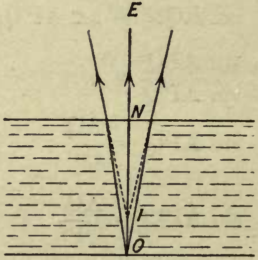

Fig. 89.

This explains why on looking vertically downwards into water the depth of the water appears to be less than it really is. Thus, Let O, in Fig. 89, represent an object at the bottom of the water, then, after refraction at the surface of the water, the small **direct** pencil incident along the normal ON as axis appears to diverge from I, so that—

$$IN = \mu \, ON.$$

But in this case μ is the index of refraction from water to air, and is equal to $\frac{3}{4}$—

and, therefore, $$IN = \tfrac{3}{4} \, ON.$$

That is, the apparent depth of the water is three-quarters of its real depth, or the image of the object at O is seen at a point I, whose distance from the surface is three-quarters of the distance of O from the surface.

In the same way the apparent thickness of a plate of glass, or other transparent medium denser than air, to an observer looking directly through the plate is less than its real thickness. For if O represent a point on the face of the plate remote from the eye, then its image as seen along the normal through O is at I, and IN is the apparent thickness of a plate of actual thickness ON. Hence, if t denote the real thickness of the plate, its apparent thickness is given by μt, where μ is the index of refraction from the material of the plate into air. For example, if the plate is of glass ($\mu = \frac{2}{3}$), the apparent thickness is about two-thirds its real thickness.

Experiment 46.—Set up the apparatus of Exp. 39, and draw the paths of the refracted rays for a small pencil of three rays (one on each side of the normal) diverging from the edge of the knife-edge at *a*, and incident directly on the front surface of the glass block. Produce the refracted rays backwards to intersect at a point on the axis of the incident pencil, and measure the distance of this point from the front surface.

The ratio of this distance to the thickness of the block evidently gives μ, the index of refraction from glass to air. The value of the ratio should be about two-thirds.

Experiment 47.—Set up the apparatus for Exp. 39 as above. Place one knife-edge, as explained in that experiment, with its edge at *a* close to the face of the block remote from the observer. Take another knife-edge, and adjust its position in front of the block until the edge of the virtual image, formed by reflection at the front face of the block, is seen to coincide (as in Exp. 17) with the knife-edge at *a*, *as seen through the block.*

Measure the distance of this knife-edge from the front surface of the block: this distance is equal to the distance of its image from the same surface, and therefore measures the apparent thickness of the block.

The ratio of the apparent thickness to the real thickness gives the refraction index from the glass of the block to air.

Experiment 48.—Fill a jug with water. Hold a strip of cardboard or paper horizontally over the surface of the water, and adjust its height above the surface until its image, formed by reflection at the surface of the water, appears, when seen through a hole in the strip, to coincide with the bottom of the jug.

Measure the distance of the strip from the surface of the water. This gives the apparent depth of the water, and the ratio of the apparent depth to the real depth gives the index of refraction from water to air.

It should be noted that the relation, $AP' = \mu AP$, is true only for the refraction of **small, directly** incident pencils.

Experiment 49.—Set up the apparatus of Exp. 39, and draw, as explained for that experiment, the refracted rays corresponding to a number of incident rays for which the angle of incidence increases from 0° (normal or direct incidence) to about 40°. Produce the directions of these rays backwards, and note how the point of intersection of adjacent rays changes as the angle of incidence increases.

63. **Image of a Point seen by Direct Refraction through a Plate.**—When a plate of glass or any transparent substance denser than air is interposed between the eye and a near object, the distance of the latter from the eye is apparently diminished.

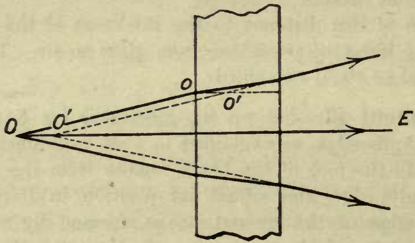

Fig. 90.

Thus, in Fig. 90, let O be a point on the object, and OE a small pencil from O refracted directly through the plate along OE to an eye at E. After refraction through the plate, the pencil evidently diverges from a point O′ in OE, and nearer to

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E than O. The distance OO′ is evidently equal to *oo′*, which corresponds to OI in Fig. 89, and is, therefore, equal to $(t - \mu t)$, where t is the thickness of the plate, and μ the index of refraction from the plate to air. If the material of the plate is less dense than air or than the surrounding medium, then it will be found on drawing the corresponding figure that O′ is further from E than O. In either case OO′, the apparent change in the distance of the object from the eye, is equal to $(t \sim \mu t)$, where t is the thickness of the plate, and μ the index of refraction from the material of the plate into the surrounding medium.

Experiment 50.—Adapt Exp. 46 to verify the result given above.

64. Refraction of a Small Pencil of Rays through a Prism in the Position of Minimum Deviation for the Axis of the Pencil.

—It has already been explained that the deviation produced by the refraction of a ray through a prism depends upon the path of the ray. When, however, the path approaches closely to the path of minimum deviation a small change in the path produces very little change in the amount of the deviation. That is, in the case of all rays passing through the prism by paths diverging slightly from that of minimum deviation, the deviation which each undergoes is practically the same, and differs very slightly from the minimum value.

Hence, if a small pencil of rays diverging from a point P (Fig. 91) be refracted through a prism by a path such that the axis follows the path of minimum deviation, then all the rays will undergo practically the same deviation and, will, therefore, be inclined to each other, after refraction through the prism, at nearly the same angle as before incidence. Hence, if the directions of the rays of the emergent pencil be produced backwards they will intersect at a point P′, such that OP′ is equal to OP.

If we imagine the direction of the light to be reversed, it will be seen that a pencil of rays, converging to P′, would, after refraction through the prism, converge to P. The points P and P′ are, therefore, conjugate foci, and if the dimensions of the prism are small compared with the distance OP, the thickness of the prism may be neglected, and it may be stated that the conjugate foci are on the same side of the prism and at the same distance from its edge.

An eye placed at E, so as to receive the pencil of rays emerging from the prism, will evidently see a virtual image of the point P at P′. In the usual case where the material of the prism is denser than the surrounding medium, this image will be displaced from P towards the refracting edge of the prism.

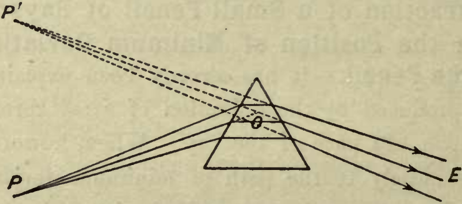

Fig. 91.

Experiment 51.—Adapt Exp. 44 to verify this result.

65. **Total Reflection.**—When a ray of light is refracted from any medium into a rarer or less dense medium, the refracted ray is bent away from the normal, and the angle of refraction is greater than the angle of incidence. It follows from this that for a certain angle of incidence *less than 90°* the angle of refraction is *equal to 90°*, and the course of the refracted ray is tangential to the surface of separation of the media at the point of emergence.

Thus in Fig. 92 the ray *ab*, in undergoing refraction from a medium, B, to a less dense medium, A, is incident on the plane surface of separation of the media at an angle, *abn*, such that the corresponding angle of refraction *nbc* is 90° and the refracted ray *bc* is, therefore, emergent along the surface of separation at *grazing* emergence.

If the angle of incidence for the ray *ab* were less than *abn* the angle of refraction would be less than 90°, and the ray would be refracted in the usual way. If, however, the angle of incidence is greater than *abn* then *the ray is not refracted at all,* but is *totally reflected* at the surface of separation of the two media. That is, for all angles of incidence less than the angle *abn* the ray *ab* is *partially* reflected and *partially* refracted in the usual way (Art. 53), but for all angles of incidence greater than the angle *abn* the ray is *totally reflected,* or suffers **total reflection** at the surface of separation of the two media. Hence, in the case of refraction from a dense to a rare medium, the angle of incidence, which corresponds to an angle of refraction of 90°, is the limiting angle at which refraction ceases and total reflection begins, and is called the **critical angle** for the two media.

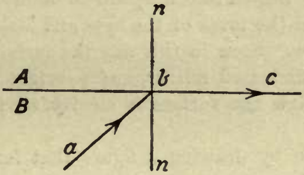

Fig. 92.

The magnitude of the critical angle for any two media is readily specified. Thus, if θ denote the angle, then the corresponding angle of refraction is, by definition, 90°, and, therefore, $\frac{\sin\theta}{\sin 90^\circ} = \mu$, where μ denotes the index of refraction *from the denser to the rarer of the two media.* But $\sin 90^\circ = 1$ and, therefore, $\sin\theta = \mu$, or the critical angle, θ, for any two media is the angle whose sine is μ, the index of refraction from the denser to the rarer of the two media. The critical angle for air and glass varies from about 38° to 41° according to the value of the refractive index, and the critical angle for air and water is about 48° 30′.

Experiment 52.—In Exp. 38 raise the right-hand end of the mirror N so as to increase gradually the angle of incidence of the ray *de* on the surface of separation of the water and air until the critical angle is reached and total reflection of the ray is seen to take place.

It will be noticed that as the angle of incidence is increased the intensity of the reflected beam *ef′* increases and that of the refracted beam *ef* decreases until, at the critical angle of incidence, the refracted ray *ef* suddenly disappears and only the reflected ray *ef′* exists; that is, the incident ray *de* is then totally reflected at the separating surface of the two media.

Experiment 53.—Place a spoon in a beaker of water. Hold the beaker above the level of the eyes and look at the surface of the water from below. Seen in this way the surface has the appearance of a brightly illuminated mirror, and a bright image of the portion of the spoon below the surface is clearly seen by reflection at the surface.

It will be seen by drawing a figure that for *all angles* at which a ray entering the eye can emerge from the side of the beaker, the direction of the ray in the water, before emergence, must strike the surface at an angle greater than the critical angle for water and air. It follows from this that the only light which can reach the eye from the surface is that which comes from objects below the surface and suffers total reflection at the surface. This explains why objects above the surface of the water, such as the handle of the spoon, cannot be seen.

Experiment 54.—Take a glass prism of which the principal section is a right-angled isosceles triangle ABC with the right angle at B (Fig. 93).

Set the prism standing on a block of wood on the table and look along a normal into the face AB. The face AC will be seen as a plane mirror, and the images of objects placed along the normal to the face BC will be clearly seen by total reflection at its surface.

It will be seen by drawing a figure that a ray of light entering the prism along a normal to the face AB is incident upon the face AC at an angle of 45°—that is, at an angle greater than the critical angle for glass and air—and is, therefore, totally reflected from this face at an angle of 45°, so that it emerges from the prism along the normal to the face BC.

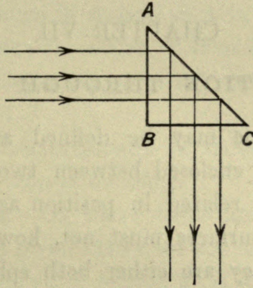

Fig. 93.

A prism of the kind described in Exp. 54 is called a **total reflection prism**; it can obviously be advantageously used, as shown in Fig. 93, to deviate the path of a beam of light through 90°.

CHAPTER VII.

REFRACTION THROUGH LENSES.

66. **Lenses.**—A lens may be defined as a portion of any transparent medium enclosed between two surfaces of regular geometrical form, so related in position as to have a common normal. The two surfaces must not, however, both be plane surfaces; usually they are either both spherical, or one plane and one spherical, and the medium most generally employed is glass.

The central cross-section of a lens enclosed between segments of two spherical surfaces is shown at LL in Fig. 94; the arc LAL represents the section of a segment having its centre at C, and the arc LA′L represents the section of a segment having its centre at C′; the line CAA′C′ joining the centres and cutting the surfaces of the lens at A and A′ is a common normal to both surfaces.

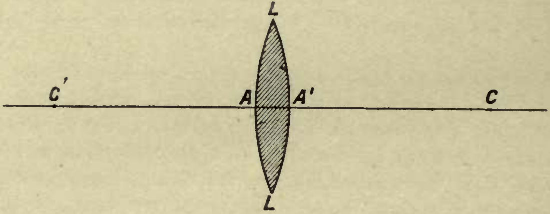

Fig. 94.

Lenses of this form may be considered as solids of revolution. For example, if a section similar to that shown in Fig. 94 be supposed to revolve round the common normal to its faces as axis, the solid described by the revolving section determines the form of the lens.

It is usual to divide lenses into two classes :—

i. **Convex Lenses.**—Of these there are three chief forms specified below.

1. *Double convex,* shown in section at A in Fig. 95.

2. *Plano-convex,* shown in section at B in Fig. 95.

3. *Concavo-convex* (*converging meniscus*) shown in section at C in Fig. 95.

The distinguishing characteristic of these lenses is that *they are thicker at the centre than at the edges.*

ii. **Concave Lenses.**—Of these there are also three forms, corresponding to those given above for convex lenses.

1. *Double concave,* shown in section at A in Fig. 96.

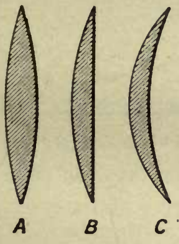

Fig. 95.

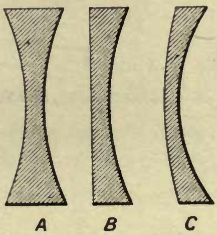

Fig. 96.

2. *Plano-concave,* shown in section at B in Fig. 96.

3. *Convexo-concave* (*diverging meniscus*), shown in section at C in Fig. 96.

The distinguishing characteristic of these lenses is that *they are thinner at the centre than at the edges.*

The action of any of these forms of lens on a pencil of rays passing through it will be found to depend on the refractive index of the material of which it is made, relative to the surrounding medium. Usually we have to deal with glass lenses surrounded by air—that is, the medium of the lens is of higher refractive power than the surrounding medium. In this case convex lenses cause the rays of a pencil to become more convergent or less divergent after passing through them, and for this reason are sometimes called **converging lenses.** Similarly, **concave lenses** are sometimes called **diverging lenses,** because the rays of a pencil of light are always under these conditions, more divergent or less convergent after refraction through them than before entering them. When the refractive index of the material of the lens is less than that of the surrounding medium, a convex lens acts as a diverging lens and a concave lens as a converging lens.

In the lenses commonly used in practice, the spherical faces are usually very small segments of the spherical surfaces of which they form part. A lens for which this condition holds, and for which the thickness is small compared with its diameter, is called a **thin lens.**

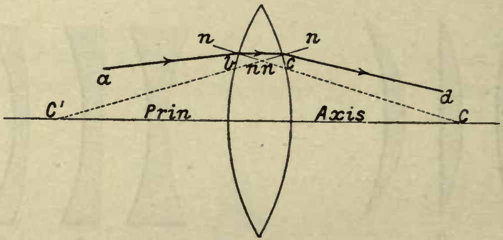

Fig. 97.

In this chapter we shall deal only with refraction through thin lenses.

67. Refraction of a Ray of Light through a Lens.—

Let AB (Fig. 97) represent a double convex lens of material denser than the surrounding medium, such as a glass lens in air. Let C and C′ be the centres of curvature of the faces of the lens, and let *ab* represent a ray incident at *b* on the face having its centre at C. The normal at *b* is readily found by drawing *nbn* in a direction passing through C, the centre of curvature of the face, and the direction of the refracted ray *bc* in the material of the lens can then be determined by the construction of Art. 57. Let this refracted ray *bc* be incident on the second face of the lens at *c*. The normal at *c* is obtained, as at *b*, by drawing *ncn* in a direction passing through C′, the centre of curvature of the second face, and the direction of the emergent refracted ray *cd* can be determined as at *b* by the same construction. In the case of a concave lens of material denser than the surrounding medium, the path of a ray, *abcd*, through the lens is determined, as shown in Fig. 98, in exactly the same way as for a convex lens.

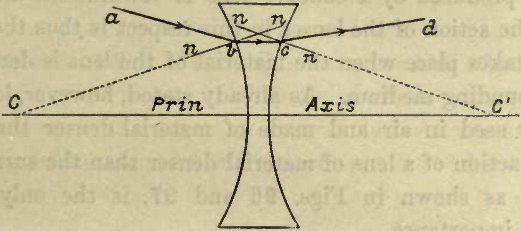

Fig. 98.

It will be noticed that when the material of the lens is denser than the surrounding medium, the ray *abcd* is deviated towards the axis of the lens by refraction through a convex lens, and away from the axis of the lens by refraction through a concave lens. When the material of the lens is less dense than the surrounding medium, the path of the ray *abcd* through a convex lens is as shown in Fig. 99, and through a concave lens as in Fig. 100. These figures are lettered to correspond with Figs. 97

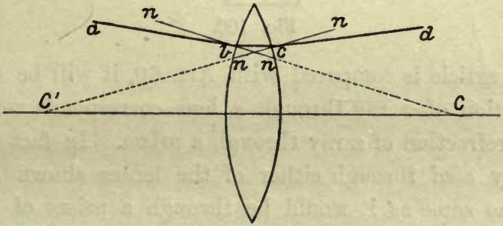

Fig. 99.

and 98, and the details of the construction in each case are exactly the same as those given above.

It will be seen from these figures that, when the material of the lens is less dense than the surrounding medium, the deviation produced by a convex lens is away from the axis of the lens, and that produced by a concave lens is towards the axis of the lens. The action of the lenses in this respect is thus the reverse of what takes place when the material of the lens is denser than the surrounding medium. As already stated, however, lenses are generally used in air and made of material denser than air, so that the action of a lens of material denser than the surrounding medium, as shown in Figs. 96 and 97, is the only case of practical importance.

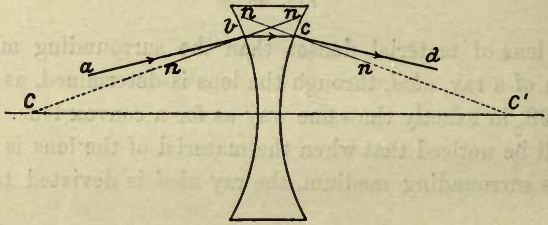

Fig. 100.

If this article is compared with Art. 60, it will be seen that the refraction of a ray through a lens corresponds very closely with the refraction of a ray through a prism. In fact the path of the ray *abcd* through either of the lenses shown above, is exactly the same as it would be through a prism of the same material having its faces tangential to the faces of the lens at the points *b* and *c*.

Experiment 55.—Arrange the apparatus for an experiment similar to Exps. 41 and 44, with a half or double convex lens instead of a plate or prism. Place the half-lens standing on the surface of section, so that the trace of one face cuts the line *ab* on the paper at *b*, as shown in Fig. 101. Now look through the lens from the other side, and place two dots *d* and *e* on the paper exactly in a line with *ab* as seen through the lens. Mark the traces of the faces of the lens, and then remove the lens. Draw a line through the dots *d* and *e*, cutting the nearest face of the lens at *c*, then join *bc*. The path *abcd* is evidently, as in the case of the plate in Exp. 41, and the prism in Exp. 44, the path by which a ray of light entering the lens along *ab* is refracted through the lens. Repeat the experiment with half a double concave lens. Note the direction of the deviation in the case of each lens.

68. **Definitions.**—The **principal axis** of a lens coincides with its axis of revolution, and, when the surfaces of the lens are spherical, passes through the centres of curvature of these surfaces. When one surface is plane and the other spherical, the principal axis passes through the centre of curvature of the spherical surface and is normal to the plane surface.

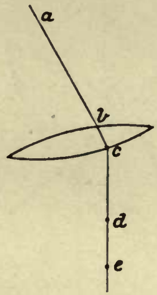

Fig. 101.

The **optical centre** of a lens is a point on the principal axis which has the property, that in the case of every ray which is refracted through the lens **without deviation**, the path of the ray within the lens passes through, or is directed through this point. That is, in the case of every ray whose direction after refraction through the lens is parallel to its direction before entering the lens, the path of the ray within the lens passes through, or is directed through the optical centre. Thus, in Fig. 102, the ray *abcd* passes through the lens AB without deviation, and the path *bc* within the lens, passes directly through the optical centre at O. In Fig. 103, however, the path *bc* within the lens does not actually pass through O, but its direction is such that if produced it passes through O—that is, it is directed through O.

It can be shown that the optical centre divides the portion of the principal axis intercepted between the faces of the lens into two segments (either internally or externally), which are directly proportional to the radii of the adjacent faces. Hence, it may be inferred that in the case of double convex or double concave lenses, the optical centre lies (Fig. 102) on the principal axis within the lens; that in the case of a plano-convex or plano-concave lens, it lies at the point where the spherical surface of the lens cuts the principal axis; and that in the case of a meniscus lens it lies (Fig. 103) on the principal axis outside the lens, on the same side as the face of smaller radius of curvature.

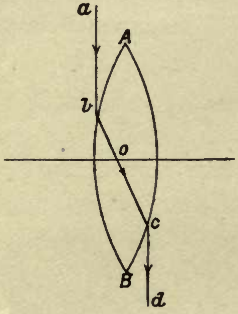

Fig. 102.

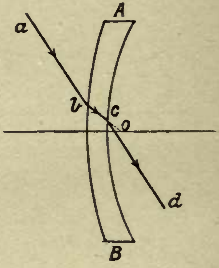

Fig. 103.

In the case of any ray which is refracted through a lens without deviation, the direction of the ray after refraction through the lens is parallel to its direction before entering the lens, but the two directions are not in the same straight line on account of the lateral displacement (Art. 59), due to the thickness of the lens. If, however, the lens is very thin this lateral displacement is negligibly small, and it follows that any straight line through the optical centre is a path of refraction, without deviation, through the lens. Further, it can be seen that even in a thick lens the optical centre lies on the principal axis, close to the faces of the lens either within or without the thickness of the lens. Hence, in the case of a **thin lens**, the position of the optical centre may be fixed without serious error by taking a point situated on the principal axis near the faces of the lens, and estimated to be approximately in the position required by the general form of the lens, as explained above.

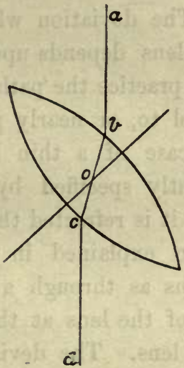

Fig. 104.

Any line, other than the principal axis, passing through the optical centre of a lens is called a **secondary axis.**

Experiment 56.—Arrange apparatus for an experiment similar to Exp. 43, with a half lens instead of a plate. Draw two parallel lines *ab* and *cd*, Fig. 104, at a distance apart equal to about half the thickness of the lens. Place the lens across the adjacent ends of these two lines, and adjust its position, as shown in the figure, until, on looking through the lens along *ab* or *dc*, the path *abcd* is seen as a continuous straight line. Now mark the trace of the lens section on the paper, cutting the parallel lines *ab* and *cd* at *b* and *c* respectively. Remove the lens and join *bc*.

The path *abcd* is obviously a path of refraction through the lens without deviation ; the portion of the path within the lens—that is, the portion *bc*, therefore passes through the optical centre of the lens, and, as this point is on the principal axis, the point *o*, where *bc* (or *bc* produced) cuts the principal axis, determines the position of the optical centre of the lens. This experiment should be repeated with several pairs of parallel lines different distances apart. It will be found in all cases for a particular lens that the *bc* part of the path always cuts the principal axis at the same point, the centre of the lens. Fairly accurate results may be obtained by this method with thick lenses. See also Exp. 63.

69. **Deviation Produced by the Refraction of a Ray through a Lens.**—The deviation which a ray undergoes on refraction through a lens depends upon the path of the ray through the lens. In practice the path of light through a lens is at all points parallel to, or nearly parallel to, the principal axis. Hence, in the case of a thin lens, the path of a ray through it is sufficiently specified by the distance from the principal axis at which it is refracted through the lens.

It has already been explained in Art. 67, that a ray is refracted through a lens as through a prism, having its faces tangential to the faces of the lens at the points where the ray enters and leaves the lens. The deviation of a ray refracted through a lens at any given distance from the principal axis is, therefore, the same as that caused by a prism of the same material as the lens, of which the refracting angle is equal to the angle between the planes tangential to the faces of the lens at the given distance from the axis. Now in the case of a thin lens this angle is small and directly proportional to the distance from the principal axis. Also, the deviation produced by a prism of small angle is directly proportional to the angle (Art. 61). Hence, in the case of a thin lens, the deviation which any ray undergoes on refraction through the lens is directly proportional to the distance of its path in the lens from the principal axis of the lens.

Thus, at a distance x from the principal axis of a double convex or double concave lens, the circular measure of the angle is approximately given by $\left(\frac{x}{r} + \frac{x}{s}\right)$ or $x\ \left(\frac{1}{r} + \frac{1}{s}\right)$, where r and s denote the numerical values of the radii of the faces of the lens. When this angle is small the deviation is given approximately (Art. 61) by $(\mu - 1)\,x\ \left(\frac{1}{r} + \frac{1}{s}\right)$, and is, therefore, proportional to x, the distance of the ray from the axis.

An important consequence of this result is that if a pencil of rays having a common focus on the principal axis is incident on a thin lens, the rays, after refraction through the lens, also form a pencil having a common focus on the principal axis.

70. **Principal Foci and Focal Length of a Lens.**—When a parallel pencil of light is incident on a lens in a direction parallel to the principal axis of the lens, the deviation which each ray of the pencil undergoes is proportional to its distance from the principal axis, and, as a result of this, it can be shown that the directions of the refracted rays all intersect at the same point on the principal axis.* This point is the **principal focus** of the lens, and its distance from the optical centre of the lens, measured along the principal axis, is the **focal length** of the lens.

In the case of a convex lens of any form, the parallel pencil of rays is made to converge to a point F (Fig. 105) on the other side of the lens. A concave lens (Fig. 106) causes the rays to diverge from a point F on the same side of the lens as the incident pencil. In each case the distance OF represents the focal length of the lens.

The focal length of a lens is independent of the direction of the rays through the lens. Hence, if F_1 (Fig. 107) be the principal focus for light travelling from left to right through a

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* If a ray at a distance x from the principal axis is deviated, so that its direction cuts the principal axis at a distance f from the centre of the lens, the deviation is approximately measured by $\frac{x}{f}$. Hence, if the deviation is directly proportional to x, the value of f must be constant for all values of x.

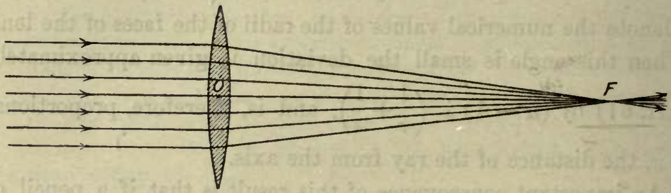

Fig. 105.

lens, then F_2, a point on the other side of the lens and at the same distance from its optical centre, is the principal focus for

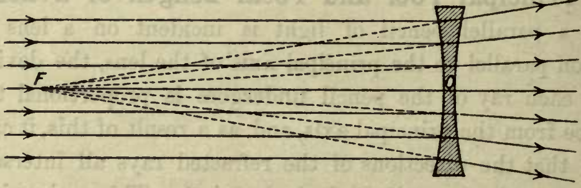

Fig. 106.

light travelling in the opposite direction through the lens. In any given case of refraction through a lens, the principal focus

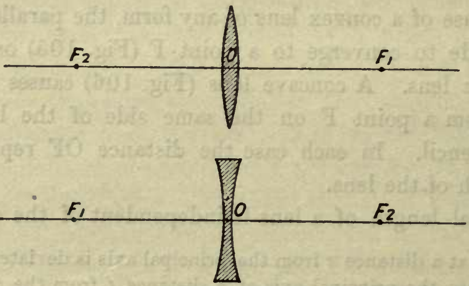

Fig. 107.

corresponding to the direction of the incident light is usually called the **first principal focus**, and the corresponding point on the other side of the lens the **second principal focus.** Thus, in Fig. 107, if the incident light travels from left to right, F_1 is the first principal focus and F_2 the second principal focus.

If a pencil of parallel light is incident on a lens in a direction parallel to a secondary axis inclined at a *small angle* to the principal axis (Fig. 108), the focus of the refracted pencil is on the secondary axis at a point F′, such that OF′ is approximately equal to the focal length of the lens.

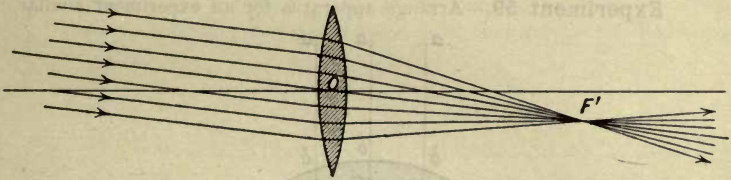

Fig. 108.

Experiment 57.—Hold a convex lens in the hand so that a beam of sunlight is incident on it in a direction parallel to the principal axis, and hold a piece of paper in the other hand as a screen to receive the beam after refraction through the lens. Hold the lens at first close up to the screen, then gradually increase the distance between lens and screen, and note how the section of the refracted beam, seen on the screen as a circle of light, changes.

It will be seen that the circle of light, at first almost as large as the lens, gradually gets narrower as the distance between the lens and the screen increases, until at a certain point, *when the distance between the screen and the lens is equal to the focal length of the lens*, it reaches its minimum size as a small and intensely bright disc (the image of the sun), and then, as the distance is still further increased, widens out again, showing that the rays cross but do not terminate at the principal focus. The heating effect at the focus of the refracted beam should also be noted.

Repeat the experiment with a concave lens. It will be found that the circle of light on the screen gets larger and less and less bright as the distance between the lens and screen increases. In order to see the circle of light in this case, it will be found necessary to mount the lens in a circular hole in a sheet of cardboard so as to keep the screen in shadow.

Experiment 58.—In a dark room place a bright flame behind a small hole in an opaque screen. Mount a convex lens on an upright stand and place it in front of the screen, so that its principal axis passes through the hole, and the distance from the hole to the lens is equal to the focal length of the lens. Now hold a second screen at different distances in front of the lens, so as to intercept the beam of light refracted through it. It will be found that the circle of light on this screen is of the same width at all points, showing that the beam is a parallel beam.

Consider the effect of substituting a plane mirror for the second screen in this experiment.

Experiment 59.—Arrange apparatus for an experiment similar to Exp. 55, with a half convex lens of short focal length. Instead of one line draw three equidistant parallel lines on the paper; let the middle line be much longer than the two outer lines, and let the distance between the latter be about one-half the diameter of the lens. Place the lens, as shown in Fig. 109, across the ends of the two outer lines, and adjust its position until, on looking through and along the middle line, this line is seen as a continuous straight line. When this adjustment is made the principal axis of the lens coincides with the middle line. Now look along each of the outer lines in turn, and in each case place two dots, *d* and *e*, exactly in a line with the line as seen through the lens. Now mark the trace of the section of the lens, cutting each of the two outer parallel lines at *b*, and then remove the lens. Join the points *d* and *e* for each of the two outer paths, and

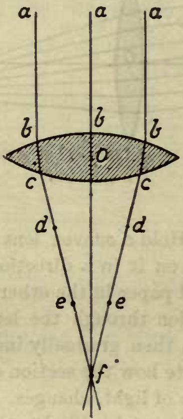

Fig. 109.

produce the lines to cut the trace of the lens section at *c* and *c* in one direction, and to cut the middle line (or principal axis) at *f* in the other direction. Then join the points *b* and *c* on each path.

It will be found that the three paths *abcdef*, *af*, and *abcdef* meet at one point, *f*, on the principal axis—that is, light entering the lens along the three parallel *ab* paths is refracted through a point, *f*, on the principal axis. This point, *f*, is the principal focus of the lens, and O*f*, the distance of this point from O, the centre of the lens, is the focal length of the lens.

Arrange a similar experiment with a half concave lens instead of a convex lens. The paths *abcde* (Fig. 110) can be determined as already explained, and the point *f* will also be found as before by producing the lines through the points *d* and *e* until they cut the middle line at *f*. In this case, however, the lines have to be produced through *c*, and the point *f* is on the same side of the lens as the parallel lines.

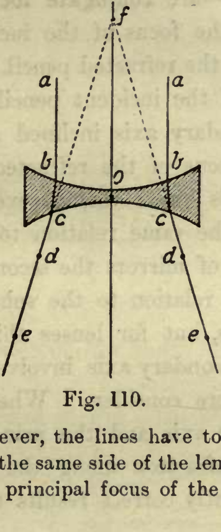

Fig. 110.

The point *f* is the principal focus of the lens, and O*f* is the focal length of the lens.

71. **Conjugate Foci.**—When a pencil of rays diverging from, or converging to, a point on the principal axis is refracted through a lens, the refracted rays also form a pencil of rays diverging from, or converging to, another point on the principal axis.* The deviation of each ray of the pencil is directly pro-

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* A parallel pencil of rays parallel to the principal axis may be considered as a pencil of rays diverging from, or converging to, a point on the principal axis at an infinite distance from the lens.

portional to the distance from the principal axis at which it is refracted through the lens, and, as a result of this, it can be shown that, in the case of a thin lens, the refracted rays must all diverge from, or converge to, a common focus on the principal axis.

Hence, when a pencil of rays diverging from, or converging to, a point on the principal axis is refracted through a lens, the refracted pencil also diverges from, or converges to, another point on the principal axis, and by the principle of reversibility the two foci—the focus of the incident pencil, and the focus of the refracted pencil—are **conjugate foci** in the sense that if either be taken as the focus of the incident pencil, the other becomes the focus of the refracted pencil.

When the focus of the incident pencil is not on the principal axis, but on a secondary axis inclined at a *small angle* to the principal axis, the focus of the refracted pencil is also on the secondary axis. It is important, however, to notice that secondary axes have not the same relation to lenses as they have to mirrors; in the case of mirrors the secondary axes have exactly the same geometrical relation to the spherical reflecting surface as the principal axis, but for lenses this is not the case, and refraction along a secondary axis involves several complications which we cannot here consider. When, however, the angle between a secondary axis and the principal axis is *small,* the laws applicable to refraction along the principal axis may be applied with sufficiently correct results to refraction along the secondary axis.

The focus of the incident pencil and the focus of the corresponding refracted pencil may be considered as **object** and **image** respectively. When the focus of the incident pencil is a point on a real object or source of light, the pencil is a divergent pencil; this is the case of greatest practical importance, and it is the only one we shall consider.

In the case of a convex lens, it will be clear from what has been said in Art. 70 that if the focus of the incident pencil is at a point F (Fig. 111) on the principal axis at a distance from the lens *equal to* its focal length, the refracted pencil will be a parallel pencil of rays parallel to the principal axis.

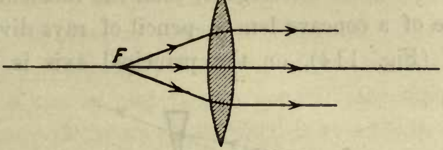

Fig. 111.

It follows from this that if the focus of the incident pencil is at a point P (Fig. 112) on the principal axis at a distance from the lens *greater than* its focal length, then the refracted pencil is a convergent pencil of rays converging to a point P′ on the principal axis on the other side of the lens. The point P′ is in this case a **real** focus. Similarly, it follows that if the focus of the incident pencil is at a point P (Fig. 113) on the principal axis at a distance from the lens *less than* its focal length, the refracted pencil is a divergent pencil of rays diverging from a point P′ on the principal axis on the same side of the lens as P. The point P′ is in this case a **virtual** focus.

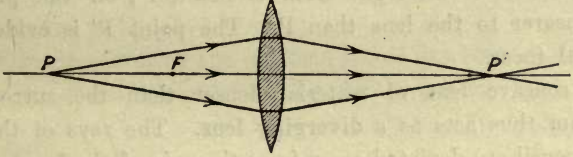

Fig. 112.

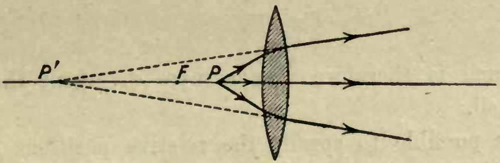

Fig. 113.

It will be noticed that in all these cases the convex lens (of material denser than the surrounding medium) acts as a **converging lens.** In each case the rays of the incident pencil are deviated towards the axis, and the refracted pencil is therefore less divergent (or more convergent) than the incident pencil.

In the case of a concave lens, a pencil of rays diverging from *any point*, P (Fig. 114), on the principal axis is made more divergent by refraction through the lens, and the refracted pencil therefore diverges from a point, P′, on the principal axis nearer to the lens than P. The point P′ is evidently a **virtual** focus.

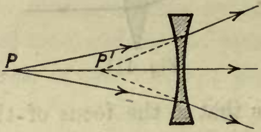

Fig. 114.

A concave lens of material denser than the surrounding medium thus acts as a **diverging lens.** The rays of the incident pencil are deviated away from the axis of the lens, and the refracted pencil is therefore always more divergent than the incident pencil.

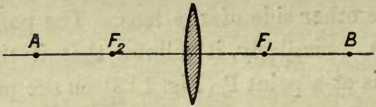

Fig. 115.

It is now possible to specify the relative positions of the conjugate foci in certain general cases.

For a convex lens the following four cases may be noted:—

1. When the incident pencil is a parallel pencil of rays parallel to the principal axis, the refracted pencil converges to F_1 (Fig. 115), the first principal focus of the lens—that is, when the

focus of the incident pencil is at infinity, the focus of the refracted pencil is at F_1.

2. When the focus of the incident pencil lies on the principal axis between infinity and F_2 (Fig. 115), the focus of the refracted pencil lies on the principal axis between F_1 and infinity on the other side of the lens, as shown in Fig. 112. This is the most important case in practice; the focus of the refracted pencil is a **real** focus. A special case of this general case should be noted. When the focus of the incident pencil is at a point A (Fig. 115) on the parallel axis *at a distance from the lens equal to twice the focal length*, the focus of the refracted pencil is at a point, B, on the principal axis at the same distance on the other side of the lens.

3. When the focus of the incident pencil is at F_2, the refracted pencil is a parallel pencil of rays parallel to the principal axis, as shown in Fig. 111.

4. When the focus of the incident pencil lies between F_2 and the lens, the focus of the refracted pencil lies, as in Fig. 113, on the same side of the lens between infinity and the lens. The focus of the refracted pencil is in this case a **virtual** focus.

From a consideration of these general cases, it is easy to follow how the **image**, or focus of the refracted pencil, moves along the principal axis, as the **object**, or focus of the incident pencil, moves along the axis from infinity up to the lens. When the object is at infinity the image is at F_1. As the object moves from infinity up to F_2, the image moves *in the same direction as the object* from F_1 to infinity on the other side of the lens. If account be taken of the fact that when the object is at a point, A, at a distance equal to twice the focal length from the lens, the image is at a point, B, an equal distance on the other side of the lens, this displacement of the object, and the corresponding displacement of the image, may be taken in two stages—that is, as the object is moved from infinity up to A, the image moves in the same direction from F_1 to B, and, as the object is moved still further from A to F_2, the image moves in the same direction from B to infinity on the other side of the lens.

As the object moves from F_2 up to the lens, the image moves in the same direction from infinity on the same side as the object up to the lens.

It should be noticed in following out corresponding displacements of image and object in this way, that *the image and object move in the same direction along the principal axis.* This is a general rule which holds good in all cases where the image is formed by **refraction**; when the image is formed by reflection it will be remembered (Art. 46) that object and image move in opposite directions along the axis of the mirror. It should also be noticed that if we compare corresponding displacements of object and image it will be found that the more distant of the two from the lens undergoes the greater displacement. Thus, in Fig. 115, as the object moves from infinity up to A the image moves only from F_1 to B; then, as the object moves from A to F_2 the image moves over the much greater range from B to infinity. It follows from this that when object and image are at A and B respectively their distance apart is a *minimum.* For, from infinity up to A the object moves faster than the image, and the distance between them decreases at first rapidly and then more and more slowly; but as the object moves from A to F_2 the image moves faster than the object, and the distance between them begins to increase, at first slowly and then more and more rapidly. The distance between the object and the image is thus a minimum when each is at a distance from the lens equal to twice its focal length, and their distance apart is, therefore, four times the focal length of the lens.

In the case of a concave lens there is only one general case for the relative position of conjugate foci as image and object. It has been explained in Art. 70 that when a parallel pencil of rays parallel to the principal axis is incident on the lens the refracted pencil diverges from F_1 (Fig. 107), the principal focus of the lens. Hence, when the object is anywhere on the principal axis between infinity and the lens the image lies between the first principal focus and the lens. That is, as the object moves from infinity up to the lens the image moves from F_1, the first principal focus, up to the lens.

A quantitative relation between the distances of conjugate foci from the lens is given in Art. 74.

Experiment 60.—Arrange for an experiment similar to Exp. 59, but draw lines diverging from a point instead of the parallel lines used in that experiment. Determine the optical continuation of each of the diverging lines by the method already explained, and note that in all cases the refracted pencil is represented by lines diverging from, or converging to, a point on the principal axis of the lens.

72. General Construction for determining the Image of an Object formed by Refraction through a Lens.—

When a pencil of rays from any point on the object is refracted through a lens the focus of the refracted pencil is the image of the point from which the incident rays diverge. Hence, if at least two rays are drawn incident on the lens from any point on the object, and the directions of the corresponding refracted rays determined, the point of intersection of the refracted rays is the image of the point on the object from which the incident rays are drawn. The refracted ray corresponding to any given incident ray could be determined by the construction of Art. 67, but this construction is much too troublesome for ordinary practice. It will be recognised from what has been said above that from any point on the object three incident rays can be drawn for which the corresponding refracted rays can be determined without any special construction. Thus, we have (1) a ray parallel to the principal axis of the lens is refracted in a direction passing through the first principal focus; (2) a ray through the optical centre of the lens passes on without deviation; (3) a ray through the second principal focus is refracted in a direction parallel to the principal axis.

The image of any point on an object is readily determined by means of any two of these three rays, and the complete image of any object is found by determining the images of a sufficient number of points on the object.

Fig. 116 shows the application of this construction for the determination of the image formed by a convex lens of a small object A B placed at a distance from the lens greater than its focal length. The images of two points, A and B, on the object are determined at A′ and B′ by means of the three rays specified above, and the image of A B as a whole is thus determined at A′ B′. The image in this case is obviously a **real** image. Similarly, Fig. 117 shows the application of the construction to the determination of the image formed by a concave lens. The image, A′ B′, of the object, A B, is in this case seen to be a **virtual** image.

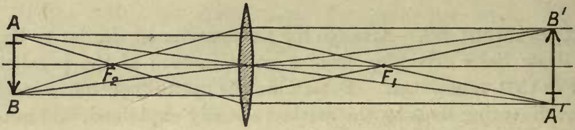

Fig. 116.

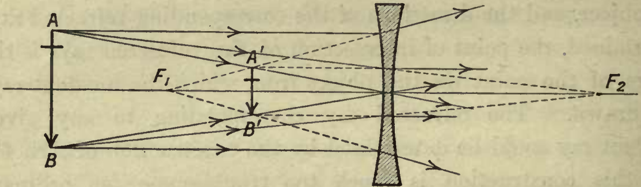

Fig. 117.

In both these figures, for the purpose of illustration, three rays are drawn from each of the points A and B. Obviously only two of the three rays are necessary for the determination of the points A′ and B′: the two rays specified under (1) and (2) above are generally used for this purpose.

73. **Formation of Images by Lenses.**—The position and character of the image formed by a **convex** lens of an object placed on the principal axis of the lens depends upon the position of the object relative to the lens.

There are three cases, corresponding to the three cases given in Art. 71, for conjugate foci, which may here be considered.

1. When the object is at infinity or, in practical terms, at a very great distance from the lens, the image is at the first principal focus.

This case is illustrated by Fig. 118. The rays coming from a point A at the top of the object form an approximately parallel pencil inclined at a small angle to the principal axis of the lens, and are therefore brought to a focus at the point A′ at a distance from the lens practically equal to the focal length of the lens.

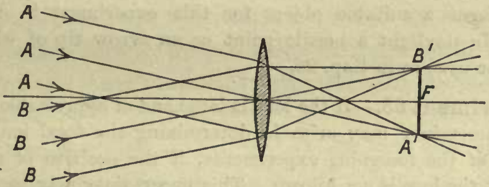

Fig. 118.

Similarly, rays from a point B at the bottom of the object are brought to a focus at the point B′, and an image, A′ B′, of the object is formed at a distance from the lens equal to its focal length.

Experiment 61.—Mount a lens on a graduated bar as in Exp. 24, with a concave mirror (or hold it in the hand as in Exp. 23), and focus an image of the sun on the screen.

Measure the distance from the lens to the screen. This gives the focal length of the lens.

Repeat the last experiment, using a distant window in the room, or a church steeple, or a tree, as object instead of the sun. The focal length found in this case will be practically the same as when the sun is taken as object.

Taking a window as object, verify that as the distance from the object to the lens decreases the distance of the image from the lens increases. In this experiment the screen should be shaded as in Exp. 57.

Experiment 62.—Set up a convex lens in front of a plane mirror with its axis at right angles to the face of the mirror. Then place an object in front of the lens and adjust its position on the principal axis until a point in the real image formed by the combined action of the lens and the mirror coincides with the corresponding point in the object, as in Exp. 26.

When this adjustment is made, the distance from the object to the lens is evidently the focal length of the lens. For, if a point in the object is at the focus of the lens, a pencil of light diverging from this point is incident on the mirror, after refraction through the lens, as a *parallel pencil normal to the surface of the mirror*, and is, therefore, reflected back, along the path by which it came, to the point from which it started.

A small candle flame, or a small hole or mark on an illuminated screen, forms a suitable object for this experiment in a darkened room. In daylight a needle point or an arrow tip of white paper should be used as in Exp. 26.

Experiment 63.—If the lens is large and of appreciable thickness, some uncertainty may arise in determining the focal length by the method of the foregoing experiments, if the position of the optical centre of the lens is not known. This uncertainty may be eliminated without finding the position of the optical centre by the following simple method.

Focus the image of the distant object on the screen, then measure (a) the distance of the screen from the pole of one of the faces, or *from any fixed point on the axis of the lens.* Now reverse the lens, and focus the distant object distinctly on the screen, and measure (b) the distance of the screen from the same fixed point on the axis as in the (a) measurement.

The mean or average of the two distances given by these two measurements is the focal length of the lens. Thus, in Fig. 119, the distance of the optical centre, O, of the lens from the screen, S, will be the same in the two positions of the lens if the image is properly focussed in each position. Now, if measurements be made to the pole A of one of the faces of the lens, and if f denote the focal length OS, and x the distance OA, then the (a) measurement gives $(f - x)$, and the (b) measurement gives $(f + x)$, and the mean of these two distances is obviously f, the true focal length of the lens.

These measurements also serve to determine the position of the optical centre of the lens, for half the difference of the measurements gives the distance x, and thus determines the distance of O from A.

This method may be used with sufficient accuracy to find the focal length of a compound lens, such as a camera lens.

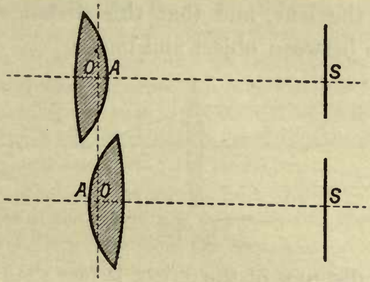

Fig. 119.

2. When the object is between infinity and the second principal focus of the lens, the image is formed on the other side of the lens, somewhere between the first principal focus and infinity.

This general case may with advantage be divided into three special cases.

(a) When the distance of the object from the lens is greater than twice the focal length, the image lies on the other side of the lens, between the first principal focus and a point at a distance from the lens less than twice the focal length.

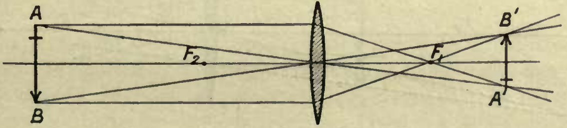

Fig. 120.

This case is illustrated by Fig. 120. It will be noticed that the image is **real, inverted, and diminished.**

(b) When the distance of the object from the lens is equal to twice the focal length, the image is formed at the same distance on the other side of the lens.

This case is illustrated by Fig. 121. The image is seen to be **real, inverted,** and **equal** to the object.

It is specially to be noticed that in this case, where the image and object are equal in size, their distance apart is four times the focal length of the lens, and that this distance is the shortest possible distance between object and image.

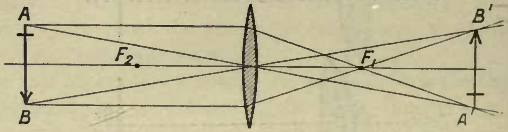

Fig. 121.

(*c*) When the distance of the object is less than twice the focal length of the lens, but greater than the focal length, the image is formed on the other side of the lens at a distance greater than twice the focal length.

This case is illustrated by Fig. 116. The image is seen to be **real, inverted,** and **magnified.**

It will be recognised that this case corresponds to the case conjugate to (*a*) above, and might be represented by reversing Fig. 120, and taking A′ B′ as the object and A B as the image.

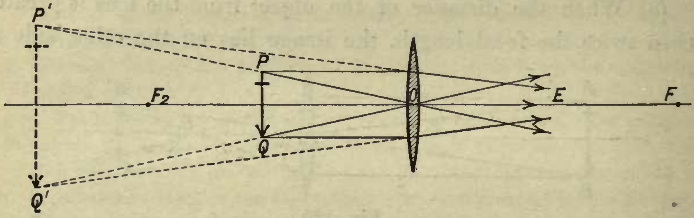

Fig. 122.

3. When the object is between the second principal focus and the lens, the image lies between infinity and the lens on the same side as the object, but at a greater distance from the lens than the object.

This case is illustrated by Fig. 122. The image is seen to be **virtual, erect,** and **magnified.** It would be seen at P′Q′ by an eye looking through the lens and placed, as at E, to receive the pencils of light from all points on PQ after refraction through the lens.

Experiment 64.—Set up an optical bench in a dark room, with a small candle flame or an illuminated screen for object, a convex lens and an image screen. Find the focal length of the lens by the method of Exp. 62, and then place the object on the principal axis of the lens at a distance from the lens (*a*) greater than twice the focal length; (*b*) equal to twice the focal length; and (*c*) less than twice the focal length, but greater than the focal length.

Focus the image on the screen in each case. In focussing notice that the edges of the image are coloured, and that the colour is blue when the screen is too far from the lens, and red when it is too near. An intermediate position can generally be found, at which the edges are practically colourless; this position may be taken as the position of correct focus. This colour difficulty, due to the different refrangibility of the different coloured constituents of white light (Art. 56) disappears if a sodium flame giving light of practically one colour (yellow) is used.

Notice that in each case, (*a*), (*b*), and (*c*) above, the image is real and inverted, and that in (*a*) it is magnified, in (*b*) equal to the object, and in (*c*) diminished in size.

In any case when the image is correctly focussed, interchange the object and image screens, and notice that a correctly focussed image of the object appears on the image screen, showing that the positions of the screens are, as **conjugate foci,** interchangeable.

Experiment 65.—Set up an optical bench with illuminated object screen, convex lens, and image screen. The object and image screens should both be ruled in small squares, about 3 mm. side, but of exactly the same size on both screens. Place the object at a distance from the lens approximately equal to twice the focal length. Focus the image on the screen, and adjust the position of the lens until the squares seen on the image coincide exactly with the squares ruled on the screen. The object and image are now exactly equal, and the distance between them is four times the focal length of the lens. Measure this distance, and divide by 4 to get the focal length.

Experiment 66.—Set up a convex lens on a table in daylight. Place a needle-point or white paper arrow-point on the principal axis of the lens at distances (*a*), (*b*), and (*c*), as in Exp. 64 above.

Locate the position of the real image in each case by means of another needle-point or arrow-point, as in Exp. 26, and note the character of the image in each case as explained in Exp. 64.

Experiment 67.—Set up a convex lens in daylight and place on the principal axis at a distance approximately equal to twice the focal length of the lens the points of a dividing compass separated by a distance of about 1 cm., and arranged so that the line joining them is at right angles to the principal axis.

Take another dividing compass with its points exactly the same distance apart, locate with this the position of the real image of the object compass points, and then adjust the position of the lens until the image points and the locating points exactly coincide. Now measure the distance between the two pairs of compass points. As in Exp. 65, one-fourth of this distance gives the focal length of the lens.

Experiment 68.—Take a convex lens and hold it at a distance less than its focal length from a printed page or any small object. A **magnified** virtual image of the print, or other object, is seen on looking through the lens, as in Fig. 122.

This experiment illustrates the use of a convex lens as a **magnifying glass.**

In the case of a concave lens the position and character of the image are the same for all positions of the object. When the object is at any point on the principal axis the image is always formed on the same side of the lens at a point on the principal axis, between the first principal focus and the lens, and nearer to the lens than the object.

This case is illustrated by Fig. 117. It will be seen that the image is always **virtual, erect,** and **diminished.**

Experiment 69.—Take a concave lens and look at any object, near or far, through it.

In all cases a virtual, erect, and diminished image of the object will be seen.

74. **Formula relating the Distances of the Object and Image from the Lens and the Focal Length of the Lens.**—A simple formula connecting these three distances, and applicable to all cases of refraction through a lens, is readily obtained by the following method.

Consider first the case of a **concave lens.** Let Fig. 123 show the construction lines for determining the image A′P′ of an object AP by a concave lens. This figure is exactly similar to Fig. 117, but shows only half the object AB and the corresponding half of the image A′B′.

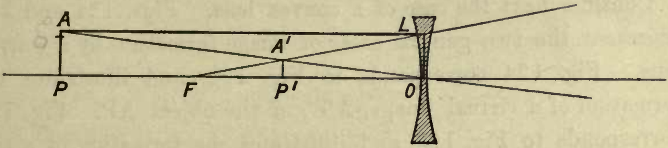

Fig. 123.

Now the triangles APO and A′P′O are similar, and, therefore,

$$\frac{\mathrm{AP}}{\mathrm{A'P'}} = \frac{\mathrm{OP}}{\mathrm{OP'}}.$$

Also the triangles FLO and FA′P′ are similar, and, therefore,

$$\frac{\mathrm{LO}}{\mathrm{A'P'}} = \frac{\mathrm{OF}}{\mathrm{P'F}}.$$

But AP is evidently equal to LO, and, therefore,

$$\frac{\mathrm{OP}}{\mathrm{OP'}} = \frac{\mathrm{OF}}{\mathrm{P'F}}.$$

Now let a, b, and c denote, respectively, the numbers measuring the lengths of OP, the distance of the object from the lens, OP′ the distance of the image from the lens, and OF the focal length of the lens. That is, a, b, and c are merely numbers measuring these lengths in any convenient unit, *without consideration of direction.* Then we have—

$$\mathrm{OP} = a,\ \mathrm{OP'} = b,\ \mathrm{OF} = c,\ \mathrm{P'F} = c - b,$$

and $$\frac{a}{b} = \frac{c}{c-b},$$

or $$ac - ab = bc,$$

and, therefore, $$ac - bc = ab.$$

Dividing throughout by abc we get—

$$\frac{1}{b}-\frac{1}{a}=\frac{1}{c}.$$

This is a numerical relation between a, b, and c for a concave lens.

Consider next the case of a **convex lens.** Figs. 124 and 125 represent the two general cases of image formation by a convex lens. Fig. 124 corresponds to Fig. 122, and illustrates the formation of a virtual image, A′P′, of the object AP. Fig. 125 corresponds to Fig. 120, and illustrates the formation of a real image, A′P′, of the object AP.

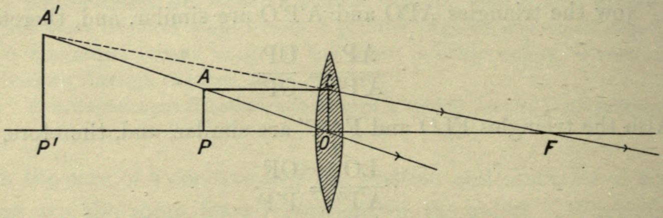

Fig. 124.

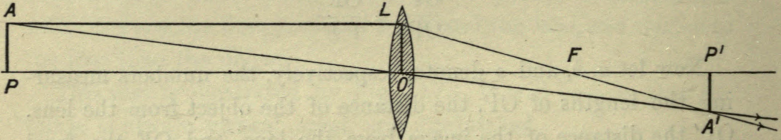

Fig. 125.

In Fig. 124, APO and A′P′O are similar triangles, and, therefore,

$$\frac{\mathrm{AP}}{\mathrm{A'P'}}=\frac{\mathrm{OP}}{\mathrm{OP'}}.$$

Also FLO and FA′P′ are similar triangles, and, therefore

$$\frac{\mathrm{LO}}{\mathrm{A'P'}}=\frac{\mathrm{OF}}{\mathrm{P'F}}.$$

But AP = LO, and, therefore,

$$\frac{OP}{OP'} = \frac{OF}{P'F}.$$

Then, adopting the same notation as above—

$$OP = a,\ OP' = b,\ OF = c,\ P'F = b + c,$$

and, therefore, $$\frac{a}{b} = \frac{c}{b + c},$$

or $$ab + ac = bc,$$

and, therefore, $$bc - ac = ab.$$

Dividing throughout by *abc* we get—

$$\frac{1}{a} - \frac{1}{b} = \frac{1}{c}.$$

Similarly in Fig. 125 we have—

$$\frac{OP}{OP'} = \frac{OF}{P'F},$$

and as $OP = a$, $OP' = b$, $OF = c$, $P'F = b - c$, we have—

$$\frac{a}{b} = \frac{c}{b - c}.$$

or $$ab - ac = bc,$$

and, therefore, $$ac + bc = ab.$$

Dividing throughout by *abc* we get—

$$\frac{1}{b} + \frac{1}{a} = \frac{1}{c}.$$

We thus get a different numerical relation for each case considered.

It is clear that it would be much more convenient to obtain a simple relation that would apply to all cases of refraction through a lens. Such a relation can be obtained by adopting the sign convention explained in Art. 50. That is, distances measured along the axis of the lens in a direction *opposed* to the incident light are to be considered *positive,* and distances measured in the same direction as the incident light are to be considered *negative.* Then, assuming this convention to hold, let u, v, and f, denote, respectively, the distances of the object, the image, and the principal focus from the centre of the lens. That is, let u, v, and f denote, not merely the numbers measuring these distances, but these numbers *each with a positive or negative sign attached according to the above convention.*

Now, on referring to Fig. 123 it will be seen that the three distances OP, OP′, OF denoted, respectively, by u, v, and f, are all positive according to this convention. Hence, remembering that a, b, and c are the numerical measures of these distances, we have—

$$u = a, \ v = b, \text{ and } f = c,$$

and the relation $\frac{1}{b} - \frac{1}{a} = \frac{1}{c}$ obtained above from Fig. 123 becomes—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

It will be found that this formula is a general algebraic relation between the three quantities u, v, and f, and is applicable to all cases of refraction through a lens.

The cases illustrated by Figs. 123, 124, and 125 differ essentially in the directions in which the lines OP, OP′ and OF are drawn from the point O, and the formula can, therefore, be made to apply to any particular case by giving the numerical values to be substituted for u, v, or f their proper signs, according to the convention just explained.

Thus, in Fig. 124 we have—

$$u = a, \ v = b, \ f = -c,$$

and substituting these values for u, v, and f in the general formula we get the relation $\frac{1}{a} - \frac{1}{b} = \frac{1}{c}$, which is the same as that obtained geometrically from the figure.

Similarly, in Fig. 125 we have $u = a$, $v = -b$, $f = -c$, and on substituting these values for u, v, and f in the general formula we get the relation $\frac{1}{b} + \frac{1}{a} = \frac{1}{c}$, which is the same as that already derived geometrically from the figure.

The relation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, associated with the sign convention given above, is thus a general relation applicable to all cases of refraction through a lens. It should be noticed that the form of the relation is the same as that of the numerical relation for the case in which all the distances involved are positive (Fig. 123) according to convention, and in proving the relation it is simplest to derive it from this case.

In applying this relation it must be remembered that all distances are measured from the centre of the lens, and that a positive or negative sign must be attached to the numerical measure of each distance according to the sign convention.

It will be seen that by this convention the measure of the focal length of a lens is positive for a concave lens, and negative for a convex lens.

The numerical relation for the important case of Fig. 124 is very commonly used. It gives $c = \frac{ab}{a+b}$, where a, b, and c are the numerical measures of the distances they denote without regard to direction. It will be seen from this relation that when $a = 2c$, then $b = \frac{ac}{a-c} = \frac{2c^2}{c} = 2c$; that is, when the distance of the object from the lens is twice the focal length, the distance of the image is also twice the focal length. It must be remembered, however, that this relation applies *only to this particular case*, whereas the u, v, f relation applies to *all* cases.

Experiment 70.—Repeat Exp. 64, and for each adjustment for image and object measure the distances u and v. Then calculate the focal length f from the relation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, or calculate c from $c = \frac{ab}{a+b}$.

Example.—When the object screen was placed at a distance of 60 cm. from the lens, the image was found to be at a distance of 90 cm. on the other side of the lens.

Here $u = 60$, $v = -90$, and f is required.

Therefore $$\frac{1}{-90} - \frac{1}{60} = \frac{1}{f},$$

or $$-\frac{5}{180} = \frac{1}{f}.$$

That is, $$f = -\frac{180}{5} = -36.$$

The focal length of the lens is thus found to be 36 cm., and the negative sign shows that it is a convex lens. Or applying the numerical relation $c = \frac{ab}{a+b}$, we get $c = \frac{90 \times 60}{150} = 36$. That is, the focal length of the lens is 36 cm.

75. Relative Size of Image and Object.—As in the case of images formed by reflection at spherical mirrors (Art. 51), so in the case of images formed by refraction through a lens, the ratio of any linear dimension of the image to the corresponding linear dimension of the object is called the **magnification** of the image, and is usually denoted by m or by writing $\frac{\text{image}}{\text{object}}$.

In Figs. 123, 124, and 125 it has already been shown that—

$$\frac{A'P'}{AP} = \frac{OP'}{OP}.$$

That is, $$\frac{\text{Image}}{\text{Object}} = \frac{\text{Distance of image from lens}}{\text{Distance of object from lens}}. \quad (1)$$

Again, from Figs. 126 and 127 it can be seen that—

$$\frac{A'P'}{LO} = \frac{P'F_1}{OF_1},$$

or, since LO = AP,

$$\frac{A'P'}{AP} = \frac{P'F_1}{OF_1}.$$

That is,

$$\frac{\text{Image}}{\text{Object}} = \frac{\text{Distance of image from first principal focus}}{\text{Focal length}}. \quad (2)$$

Also, from the same figures we get—

$$\frac{\text{MO}}{\text{AP}} = \frac{\text{OF}_2}{\text{PF}_2},$$

or, since MO = A′P′,

$$\frac{\text{A}'\text{P}'}{\text{AP}} = \frac{\text{OF}_2}{\text{PF}_2}.$$

That is,

$$\frac{\text{Image}}{\text{Object}} = \frac{\text{Focal length}}{\text{Distance of object from second principal focus}}. \quad (3)$$

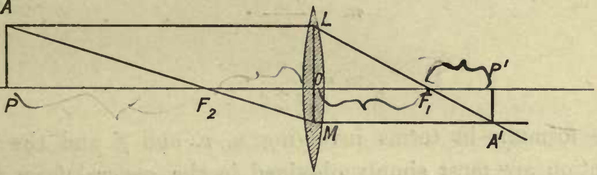

Fig. 126.

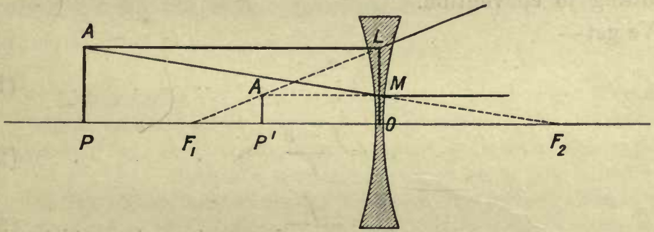

Fig. 127.

It should be noted that in all these relations, as in the corresponding relations for images formed by reflection at spherical mirrors, the distance of the image from any point when it occurs in the relation always appears as the first term of the magnification ratio, and the distance of the object from any point when it appears is always found as the second term of the ratio.

These relations are so easily deduced from the figure corresponding to any given case that they need scarcely be committed to memory, but, if they are, they are best learnt in words, as given above, and applied to any given case from a figure representing the case.

The relations are, however, readily expressed as arithmetical formulæ in terms of a, b, and c, as in Art. 74, or in algebraic formulæ involving u, v, and f, and the sign convention, as explained above.

Thus, in terms of a, b, and c we have—

$$m = \frac{a}{b}. \quad . \quad . \quad . \quad . \quad (1)$$

$$m = \frac{b \overset{+}{\backsim} c}{c}. \quad . \quad . \quad . \quad (2)$$

$$m = \frac{c}{a \overset{+}{\backsim} c}. \quad . \quad . \quad . \quad (3)$$

The formulæ in terms involving u, v, and f and the sign convention are most simply obtained in the general form from Fig. 127, where all the distances concerned are positive, according to convention.

We get—

$$m = \frac{v}{u}. \quad . \quad . \quad . \quad . \quad (1)$$

$$m = \frac{f - v}{f}. \quad . \quad . \quad . \quad (2)$$

$$m = \frac{f}{u + f}. \quad . \quad . \quad . \quad (3)$$

In this form the formulæ are applicable to any case of refraction through a lens, provided it is remembered that u, v, and f denote here, as in Art. 74, the numbers measuring the distances of the object, the image, and the first principal focus **from the lens**, with a positive or negative sign attached to indicate direction according to the sign convention.

When the object and image are on opposite sides of the lens, the image is always inverted; the rays from the top and bottom of the object cross in passing through the optical centre of the lens.

Hence, in these formulæ, if v and u are of the same sign, m is **positive** and the image is **erect**; if v and u are of opposite sign, m is **negative** and the image is **inverted.**

If the distance of the image from the **first** principal focus be denoted by x', and the distance of the object from the **second** principal focus by x, we evidently get from relations (2) and (3) above—

$$m = \frac{x'}{f} = \frac{f}{x}.$$

Therefore, $$\frac{x'}{f} = \frac{f}{x},$$

or $$xx' = f^2.$$

This formula may be taken numerically, or subject to the sign convention. It corresponds to the similar formula obtained in Art. 51 for reflection at a spherical mirror. It must be noticed, however, that in this case x and x' are measured from different points, x from the second principal focus and x' from the first principal focus.

Experiment 71.—Set up an optical bench with illuminated object screen, convex lens, and image screen. The object screen should have three or four concentric circles drawn on it as explained in Art. 49. Place the lens in position so as to give a real image of the object, and focus this image on the screen. Measure the diameter of one or more of the image circles focussed on the screen, and calculate the magnification of the image. Measure also the distances of the object and image from the lens, and then—

i. Verify the relation $m = \frac{v}{u}$.

ii. Find the focal length of the lens from the relation $m = \frac{f - v}{f}$.

iii. Find the focal length from the relation $m = \frac{f}{u + f}$.

iv. Find the focal length from the relation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$.

Note that if m is known, f can be calculated from a knowledge of either v or u.

Example.—In an experiment similar to that suggested above, the following measurements were made:—

An object, placed at a distance of 50 cm. from the lens, gave a sharply defined, real, inverted image on the screen at a distance of 66·7 cm. on the other side of the lens. The circles on the object screen were 10 mm., 15 mm., and 20 mm. in diameter, and the diameters of the corresponding circles on the image were found to be 13·5 mm., 20 mm., and 26·4 mm., respectively.

From these data we get—

$$m = \frac{13{\cdot}5}{10} = 1{\cdot}35.$$

$$m = \frac{20}{15} = 1{\cdot}33.$$

$$m = \frac{26{\cdot}4}{20} = 1{\cdot}32.$$

The mean value is, therefore, given by $m = 1{\cdot}33$.

If we adopt the sign convention formulæ we must, since the image is inverted, write $m = -1{\cdot}33$.

Also we have—

i. $m = \frac{v}{u} = \frac{-66{\cdot}7}{50} = -1{\cdot}33.$

ii. From $m = \frac{f-v}{f}$ we get $-1{\cdot}33 = \frac{f+66{\cdot}7}{f}$,

and, therefore, $f = -28{\cdot}6$ nearly.

iii. From $m = \frac{f}{u+f}$ we get $-1{\cdot}33 = \frac{f}{50+f}$,

and, therefore, $f = -28{\cdot}6$, nearly.

iv. From $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we get $\frac{1}{-66{\cdot}7} - \frac{1}{50} = \frac{1}{f}$.

and, therefore, $f = -28{\cdot}6$ nearly.

That is, the focal length of the lens is nearly 28·6 cm. The sign attached to f is negative as it should be for a convex lens.

76. **Spherical Aberration in a Lens.**—It has already been explained (Art. 71), that when a pencil of rays diverging from or converging to a point on the principal axis is refracted through a lens, the refracted rays also form a pencil diverging from, or converging to, *a point* on the principal axis. This is

approximately true (Art. 69), when the faces of the lens are very small segments of the spherical surfaces of which they form part, but it is far from true if the faces are comparatively large segments of spherical surfaces. In all cases the deviation of the rays increases with increase of distance from the principal axis *more* than is necessary to bring all the refracted rays to a focus at one point on the principal axis; and, as a result, the focus of rays refracted through the lens at a given distance from the axis gets nearer to the lens as this distance increases. Thus, in Fig. 128, the focus of the outer rays of the parallel pencil refracted through the lens is nearer the lens than the focus of the central rays of the pencil.

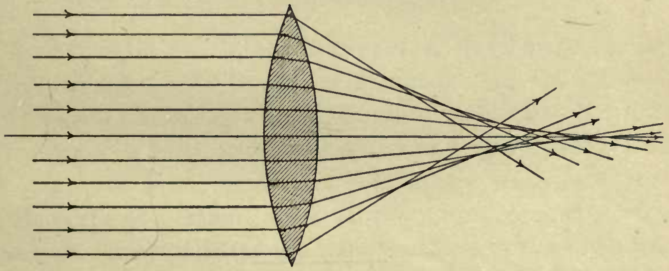

Fig. 128.

This defect is due to the spherical form of the faces of the lens, and is known as **spherical aberration.** It is inappreciable when the faces of the lens are small segments of the spherical surfaces of which they form part. The defect may be remedied in any lens by interposing a **stop** or diaphragm with a small circular hole in it in front of the lens, so as to allow rays to pass only through the central portion of the lens.

Experiment 72.—Take a large convex lens of short focus, and set it up so that it gives a real image of a candle flame on a screen. Notice that the image is blurred and cannot be clearly and sharply focussed.

Now place in front of the lens a sheet of cardboard with a small hole in it, and adjust the hole directly in front of the centre of the lens. It will now be found that the image can be clearly and sharply focussed by properly adjusting the position of the screen. Make this adjustment, then remove the cardboard diaphragm and replace it by a disc of cardboard, about half the diameter of the lens, placed so as to cover up the central portion of the lens. Refraction now takes place through a marginal ring of the lens, and it will be found that the image on the screen is badly out of focus, but can again be focussed clearly by moving the screen *nearer to* the lens.

CHAPTER VIII.

DISPERSION.

77. **The Composite Character of White Light.**—It has already been explained that the light given out by any source is, in general, of a composite character. That is, the wave motion emitted by the source is not, in general, simple wave motion of a particular wave length, but compound wave motion made up of many components or constituents of different wave lengths.

As stated in Art. 4, the difference of wave length in light waves is perceived by the eye as difference of colour. A simple light wave of definite wave length, when incident on the retina, gives rise to a particular colour sensation which is characteristic of that wave length, and a compound light wave, made up of components of different wave lengths, gives rise to a colour sensation compounded of the sensations due to the components of which it is made up.

The light from a *coloured* source of light may, therefore, in theory be either simple or compound in character. No source of light of one particular wave length—that is, no source of *homogeneous* or *monochromatic* light—is known in practice. The light emitted by the incandescent vapour of sodium is, however, approximately monochromatic, for it is made up of a few constituents whose wave lengths lie within a narrow range corresponding to slightly different shades of the same colour—yellow.

A *sodium flame*—that is, an ordinary Bunsen flame coloured yellow by a bead of any sodium salt—is, therefore, generally used as a convenient source of practically monochromatic light.

The action of the flame on the bead of salt (fused common salt, or sodium chloride, is generally used) produces sodium vapour in the flame when it is heated to incandescence, and gives rise to the yellow colour characteristic of the flame.

White light from an incandescent source at a *white* heat (Art. 4) is compound in character, and is made up of an infinite number of constituents of *all* wave lengths within the range to which the eye is sensitive.

The composite character of the white light from the sun was first demonstrated experimentally by Newton, in 1676, by a famous experiment now generally known as *Newton's experiment.*

Newton performed this experiment in a carefully darkened room from which the sunlight was excluded by means of wooden shutters. A hole, O, about one-third of an inch in diameter, was made in one shutter so as to admit a narrow beam of sunlight into the room. This beam, falling on the opposite wall of the room, produced on the wall an elliptically shaped image of the sun, about two inches in horizontal diameter, as shown at Q in Fig. 129. This image was formed in the same manner as the image of an external object is formed in a pin-hole camera. A glass prism, P, was then placed in the path of this beam of light, with its length horizontal and at right angles to the vertical plane containing the beam. The refracting edge of the prism was turned downwards so that the beam of sunlight was refracted upwards away from the edge, and produced a

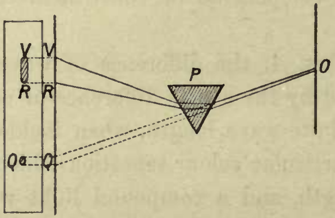

Fig. 129.

refracted image of the sun on the wall at RV. Before finally fixing the prism in position it was rotated slowly round a horizontal axis, parallel to its refracting edge, so as to lower the position of the image RV on the wall, and bring it as near as possible to its first position at Q. In this way the prism was fixed in the position of minimum deviation (Art. 61) for the beam of light incident on it, and the image at RV was thereby made as clear and as well defined as possible.

A sheet of white paper was fixed on the wall to receive the image at RV, and the image was then examined. It was found to be oblong in shape, about 10 inches long and 2 inches wide, and bounded by two straight parallel sides and semicircular ends, as shown in Fig. 129 above. The most important characteristic, however, was the colour of the image. It was found to be red at its lowest or least refracted end R, violet at its highest or most refracted end V, and between the ends its colour changed from red to violet through various shades of orange, yellow, green, and blue in passing from the end R to the end V.

The explanation of Newton's experiment is evidently found in the facts that white sunlight is composite in character, and that the constituents of different wave lengths, corresponding to different colour sensations, are refracted to different extents. The red light waves of longest wave length are refracted least, and the violet light waves of shortest wave length are refracted most, while between these limits the waves of gradually decreasing wave length, corresponding to the different shades of red, orange, yellow, green, blue, and violet, which constitute white sunlight, are refracted to extents which increase as the wave length diminishes.

Newton expressed these facts in his *Opticks* by saying that "lights which differ in colour, differ also in degrees of refrangibility," and that "the light of the sun consists of rays differently refrangible."

The term *refrangibility* here used evidently applies to the degree of deviation which is produced by refraction under given conditions. Thus the different coloured constituents of white light are deviated to different extents by refraction through the prism in Newton's experiment, and are, therefore, said to be "differently refrangible," or of different refrangibility. The violet light of short wave length is deviated to a greater extent than the red light of longer wave length, and is said, therefore, to be more refrangible, or of higher refrangibility than red light. That is, the refrangibility of light increases as its wave length decreases.

Newton showed by direct experiment that violet light is more refrangible than red light. The coloured image RV (Fig. 129), obtained as explained above, was allowed to fall on a screen with a small hole in it. The screen was first adjusted so that the red end of the image fell on the hole and a beam of red light passed through the hole beyond the screen. This beam was refracted through a prism and the minimum deviation produced by the prism was noted. The screen was then adjusted so as to allow a beam of violet light to pass through the hole, and the minimum deviation produced in this beam by the same prism was noted. It was found that the beam of violet light was deviated to a greater extent than the beam of red light. That is, blue light is more refrangible than red light, or, as Newton puts it, "light which being most refracted in the first prism did go to the blue end of the image, was again more refracted in the second prism than the light which went to the red end of the image."

78. **A Pure Spectrum.**—A coloured image of the type obtained at RV (Fig. 129), in Newton's experiment is called a *spectrum*. The characteristic of the image is that it shows by the distribution of colour from one end to the other the constitution of the light from which it is derived.

The spectrum obtained in Newton's experiment is not, however, a *pure spectrum*, for the colour at any point, except the extreme ends, does not correspond to any single constituent of the sunlight incident on the prism, but to a combination of a number of these constituents. The beam of sunlight is separated by refraction through the prism into an infinite number of constituent beams of different colours. The separation thus produced between any two adjacent beams is, however, very small, so that, unless the beams are very narrow, they must overlap considerably at the point where they fall upon the screen. It follows from this that the colour at any point on the image is a combination of the colours of the beams which overlap at that point, and is not the pure colour corresponding to a particular constituent of the sunlight.

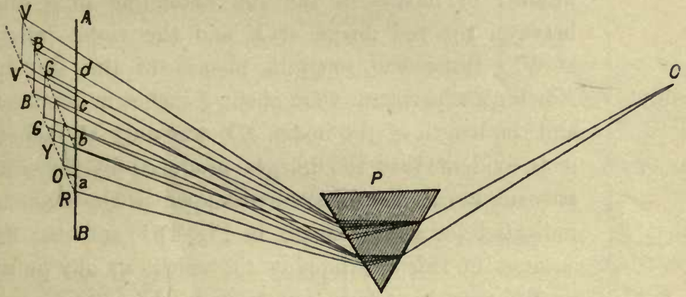

Fig. 130.

The extent to which this overlapping takes place under the conditions of Newton's experiment is shown in Fig. 130. In this figure the lines RR, OO, YY, GG, BB, and VV indicate the width and relative positions of the constituent beams corresponding to certain shades * of red, orange, yellow, green, blue, and violet. It will be seen that on account of the width of the beam and the small angular separation of the constituents there is much overlapping, and the colour at any point on the screen at AB is far from pure. Thus, at *a* the colour is evidently made up of many shades of red; at *b*, of some shades of red, all shades of orange, and some shades of yellow; at *c*, of some shades of yellow, all shades of green, and some shades of blue; and at *d*, of several shades of blue and all shades of violet.

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* The shade corresponding to the longest wave length for each colour.

The character of the spectrum given by Newton's experiment may also be realised by noting that just as the beam of sunlight gives a white image of the sun at Q, Fig. 129, before the interposition of the prism so, after refraction through the prism, each constituent of the refracted beam should give a coloured image of the sun on the screen. The image RV, Fig. 129, is, therefore, a composite image made up of an infinite number of images of the sun extending in a line between the red image at R and the violet image at V. Since the separate images of the sun in Newton's experiment were about 2 inches in diameter and the length of the image RV was only 10 inches, it is evident that the images produced by the constituent beams must have overlapped in the manner indicated for a few images in Fig. 131, and that on account of this overlapping the colour at any point on RV must have been composite in character.

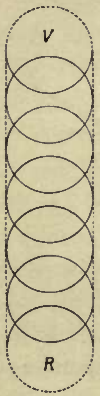

Fig. 131.

It will now be understood, from what has been said above, that in order to obtain a pure spectrum by refraction of a beam of composite light through a prism, there must be no over-lapping of the constituent beams at the point where they are incident on the screen after refraction through the prism. The beam of light incident on the prism must, therefore, be of very small width (in the plane of incidence), and should for that reason be derived from a very narrow slit placed with its length parallel to the refracting edge of the prism.

It is evident, too, that if the beam is divergent its width increases as the distance from the source increases, so that, even if it is narrow at the source, its width at the screen after refraction through the prism may be great enough to cause considerable overlapping of the constituent beams. The most convenient method of avoiding this difficulty is to place a convex lens in the path of the beam at a point where it will bring the refracted beams emergent from the prism to a focus on the screen.

It has been explained in Art. 64 that if a pencil of monochromatic light diverging from a point, O (Fig. 132), is refracted through a prism, *in the position of minimum deviation* for the axis of the pencil, the emergent pencil diverges from a virtual focus at O′. Hence, if a convex lens is placed in the path of the emergent pencil at L, it will bring the pencil to a focus at a point, Q, conjugate to O′. Similarly, if the lens is placed in the path of the incident pencil, as at L in Fig. 133, so that the pencil incident on the prism, in the position of minimum deviation, converges to a point, Q′ conjugate to O behind the prism, then the emergent pencil, after refraction through the prism, will converge to a focus at Q.

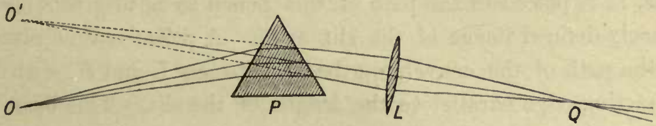

Fig. 132.

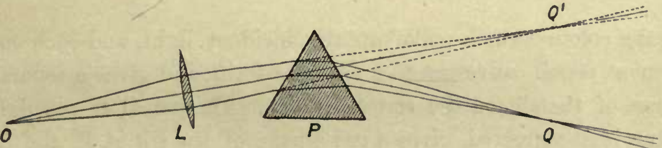

Fig. 133.

It is to be noted that in both these arrangements the prism must be placed in the position of minimum deviation for the axis of the pencil, for if the prism is not in this position when a divergent or convergent pencil of rays is refracted through it, the rays of the emergent pencil, after refraction through the prism, do not all pass through a common focus.

The usual methods of obtaining a pure spectrum will now be readily followed. The method indicated in Fig. 134 is based on the arrangement shown in Fig. 133.

A divergent pencil of the light from which the spectrum is to be derived is admitted through a very narrow slit, S, and a convex lens, L, is placed in the path of this pencil so as to give a clear, sharply-defined image of the slit at S′. A prism is then placed in the path of the convergent beam between L and S′, with its refracting edge parallel to the length of the slit. This beam is separated by refraction through the prism into constituent pencils of the colours which make up the incident light, and each constituent pencil converges to a separate focus, and gives a separate image of the slit on the screen at AB. The pencil of red light, being least refracted, gives a red image of the slit at R, and the pencil of violet light, being most refracted, gives a violet image of the slit at V; the intermediate pencils, being more refrangible than red, and less refrangible than violet, give coloured images of the slit between R and V.

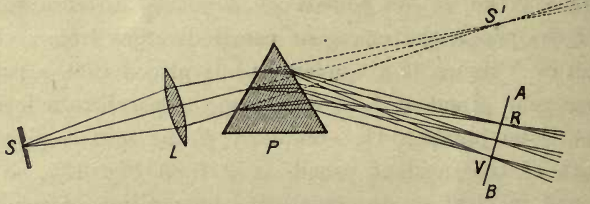

Fig. 134.

All these images are very narrow lines or bands of colour, so that there is very little overlapping of adjacent images, and the spectrum is practically a pure spectrum, showing at any point the pure colour corresponding to one particular constituent of the incident light.

A modification of this method, corresponding to the arrangement of Fig. 132, is shown in Fig. 135. The lens L is placed in the path of the emergent pencil after refraction through the prism, and the constituent pencils, corresponding to the constituent colours of the spectrum, are brought to a focus on a screen placed at AB. A spectrum made up of line images of the slit is thus obtained on the screen at RV, as already explained.

In both these methods the prism is placed in the position of minimum deviation *for the axis of the pencil.* When the pencil is small (Art. 64) this arrangement is satisfactory, but when the pencil is fairly large the different rays (of the same colour) suffer appreciably different deviations by refraction through the prism, and the rays of the emergent pencil cease to have a common focus. The images which make up the spectrum thus become blurred and indistinct if the incident pencil is too large. On the other hand, if the incident pencil is sufficiently small to give well-defined images, the spectrum may not be bright enough. In order to avoid this difficulty, and to obtain a bright and well-defined spectrum, the following arrangement is generally used.

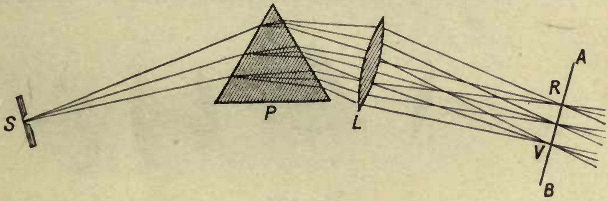

Fig. 135.

A convex lens, L (Fig. 136), is placed in the path of the incident beam at a distance equal to its focal length from the slit S. The pencil, after refraction through the lens, thus becomes a *parallel pencil,* and when this parallel pencil is incident on the prism the angle of incidence is the same for every ray of the pencil.

The pencil is separated by refraction through the prism into its constituents, and, as every ray of the same colour is deviated to the same extent, the constituent pencils of the emergent beam are all parallel pencils. Although each constituent pencil is thus a parallel pencil, the constituent pencils are not parallel to each other, for the deviation of each pencil depends upon its wave length and increases as the wave length diminishes. Hence, if a convex lens, M, is placed in the path of the emergent beam each constituent pencil is brought to a separate focus on a screen, AB, placed at the principal focus of the lens. The pencil of red light of longest wave length, being deviated least, is brought to a focus at R, and the pencil of violet light of shortest wave length, being deviated most, is brought to a focus at V. Between these limits the intermediate pencils are brought to a focus at points between R and V, and a pure spectrum of the incident light is obtained on the screen.

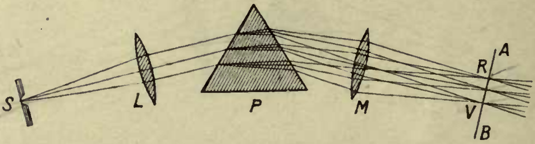

Fig. 136.

The methods described above show how to obtain a *real* spectrum which can be thrown on a screen. A *virtual* pure spectrum of any source of light can be seen by looking through a prism at a narrow slit illuminated by the source. The prism is placed with its edge parallel to the slit and its position is adjusted, as shown in Fig. 137, so that the divergent pencil of light from the slit S is refracted through it in the position of minimum deviation for the axis of the pencil. This pencil is separated by refraction through the prism into constituent pencils, each of which diverges from a small *virtual image* of the slit at a point between R and V in the figure. The pencil of red light is least deviated and diverges from the point R, and the pencil of violet light is most deviated and diverges from V. Between these limits the intermediate pencils diverge from points between R and V, so that an eye placed at E to receive the emergent pencil sees a virtual spectrum of the slit at RV. The pencil refracted through the prism from the slit at S to the eye at E is, in this case, very small, so that if the prism is in the position of minimum deviation for the axis of the pencil, each of the constituent coloured pencils which enters the eye diverges from a narrow, well-defined virtual image of the slit, and the spectrum seen is, therefore, a pure spectrum.

If the slit is not sufficiently narrow the images which make up the spectrum may be wide enough to overlap appreciably, and so spoil the definition and purity of the spectrum. When the slit is very wide the images overlap to such an extent that the spectrum becomes a white image of the slit, with a coloured border edged with red on one side, and a coloured border edged with blue on the other side.

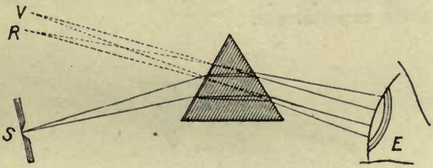

Fig. 137.

Experiment 73.—Hold a glass prism with its refracting edge parallel to a window bar, and look at the bar through the prism. Notice that one side of the image is edged with red, and the other with blue or violet.

Try the same experiment with a candle flame in a dark room.

Cut a wide slit in a sheet of cardboard, and place it against a window. Then look at this slit through a prism held with its edge parallel to the length of the slit. Try the same experiment with a much narrower slit, and then with a very narrow slit. Compare the character of the image seen in each case with the explanations given above.

Experiment 74.—Cut a very narrow slit in the middle of a sheet of cardboard, and set up the sheet in a dark room with a bright white light behind it to illuminate the slit. Then set up a prism in front of the sheet with its edge parallel to the slit, and adjust its position, as shown in Fig. 137, until a pure virtual spectrum of the slit is seen on looking through the prism at the slit along the path of minimum deviation.

Note the character of the spectrum, and then replace the white light behind the slit by a bright sodium flame. Note that the spectrum is now reduced to a very narrow yellow image of the slit. This image is really a very short spectrum extending over a very short length in the yellow part of the white light spectrum.

Try the effect of increasing the width of the slit in this experiment with both sources of light.

A slit of adjustable width between metal knife edges will be found useful for these experiments.

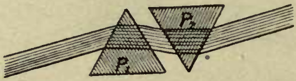

Fig. 138.

When a spectrum is obtained, as explained in this article, by refraction through a prism, it is sometimes called a *prismatic spectrum*, and the spectrum of sunlight is generally known as the *solar spectrum*.

If the colours which constitute the spectrum of any source of light are recombined in any way, it is found that they reconstitute the light from which they are derived.

Thus, if a parallel pencil of white light, after refraction through a prism, P_1 (Fig. 138), is refracted through an exactly similar prism of the same material placed in the reversed position, shown at P_2 in the figure, the pencil emerging from this second prism is a parallel pencil of white light similar to the incident pencil. The *angular separation* produced between the constituents of the light by refraction through the first prism is done away with by refraction through the second prism, so that the constituents of the pencil emerging from this prism are parallel to each other, and reconstitute white light as in the incident pencil. The two prisms combined in this way are, in fact, equivalent to a *plate* of the same material, and the effect of refraction through the combination is the same as that of refraction through a plate: the rays of the emergent pencil are parallel to the rays of the incident pencil, and, therefore, to each other. It is to be noticed, however, that the constitution of the emergent pencil is not exactly the same as that of the incident pencil. The constituents of the incident pencil are refracted to different extents on entering the plate so that angular separation is produced by this first refraction into the plate. This angular separation is, however, destroyed by the second refraction out of the plate, for the emergent pencils of all colours are parallel to the incident pencil, and parallel, therefore, to each other. The constituents of the emergent pencil have, however, suffered different lateral displacements (Art. 59) by refraction through the plate, so that they are separated by small lateral displacements relative to each other. This is shown in Fig. 139, where RR indicates the position of the red pencil and VV the position of the violet pencil in the emergent beam. Between these two pencils and parallel to them lie all the other constituent pencils in order of refrangi-

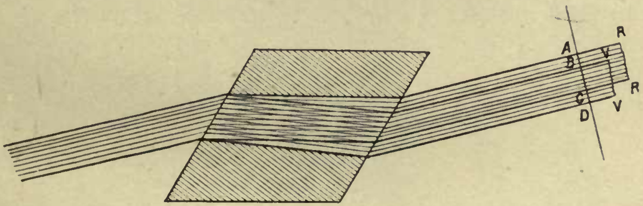

Fig. 139.

bility. Hence, if this emergent beam is thrown on a white screen at ABCD, the central position, where *all* the constituent pencils overlap, will be found to be white, but the edges AB and CD will be coloured. The edge AB shades off from red at A to white at B, and the edge CD from violet at D to white at C, the colour at any point in each case being the colour produced by the combination of the colours of the constituent pencils which overlap at that point.

79. **Dispersion.**—When a ray of composite light is refracted from one medium into another, its constituent rays are deviated to different extents and so undergo angular separation at the point of incidence. This angular separation between the constituents of a ray of composite light, produced by refraction of the ray from one medium into another, is called **dispersion.** Thus, when a ray of white light, PQ (Fig. 140), is refracted from the medium A into the medium B, the red light constituent is refracted along QR, and the more refrangible violet constituent along QV. The angular separation thus produced between these two constituents is measured by the angle RQV, and this angle is said to be the dispersion produced between the red and violet constituents of white light by the refraction of the ray PQ from the medium A to the medium B. Similarly, the dispersion produced between any two constituents of the incident ray is given by the angle between the refracted rays corresponding to these constituents. If the ray QR corresponds to the

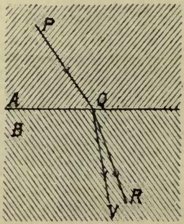

Fig. 140.

red constituent of longest wave length, and QV to the violet constituent of shortest wave length, then angle RQV is said to be the *extreme dispersion*, sometimes *the dispersion*, produced by this particular case of refraction.

It will be clear that the dispersion produced between any two constituents of a ray of composite light in any given case of refraction depends upon the media involved and also upon the angle at which the composite ray is incident upon the surface of separation of the media.

The dispersion produced by refraction through a prism is a case of some importance. The dispersion for any two constituents is given in this case by the angle between the two constituent rays in the emergent pencil. Thus, in Fig. 141 the dispersion for the two constituents QAB and QCD of the incident ray PQ, is given by the angle BOD between the two constituent rays AB and CD of the emergent pencil.

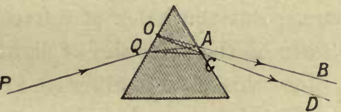

Fig. 141.

When it is desired to associate the dispersion produced by refraction through a prism with the path of minimum deviation through the prism it is usual to take the dispersion for any two constituents of the incident ray to be given by the difference of the minimum deviations for these constituents. Thus, if D_1 denote the minimum deviation for the more refrangible of the two constituents, and D_2 the minimum deviation for the other, then $(D_1 - D_2)$ gives the minimum deviation dispersion for these constituents.

If two prisms of different materials are constructed with different angles, so as to give the same minimum deviation for any given constituent, it will be found that, in general, they give quite different deviations for every other constituent. The dispersion or difference of the deviations for any two constituents of white light will thus, in general, be different for prisms of different materials, even when the deviation of one of the constituents is the same for both prisms. That is, the dispersion produced between any two constituents under given conditions depends ultimately upon the material of the prism.

The dispersion which accompanies refraction from one medium to another shows that in the case of some media light of different colours must travel with different velocities in the same medium. Since the coloured constituents of white light are refracted to different extents when a ray of white light is refracted from one medium to another, it follows that the relative index of refraction for the two media must be different for different colours. But the index of refraction for any colour (Art. 56) is the ratio of the velocities of light of that colour in the two media. Since this ratio is different for different colours, it follows that different coloured lights must have different velocities in, at least, one of the two media.

Light of all colours appears to travel with the same velocity in free space (ether) and in air, for if this were not the case a star would appear coloured just before and after suffering eclipse. Before eclipse the last colour seen would be that of the light which travels with lowest velocity, and after eclipse the first colour seen would be that of the light which travels with greatest velocity. No such colour effects have been observed, and it is concluded, therefore, that light of all colours travels with the same velocity in ether and in air.

When, however, white light is refracted from air into another denser medium, such as water or glass, the different coloured constituents are refracted to different extents, and it follows, therefore, as explained above, that the velocity of light in this medium must be different for different colours. Thus, violet light is refracted to a greater extent than red light, and since the velocity is the same for these two colours in air, it follows that the velocity of violet light in the second medium must be less than that of the red light. For example, the refractive indices of crown glass for red and violet light are about 1·51 and 1·53, respectively, so that the velocity of red light in crown glass is about $1 \cdot 99 \times 10^{10}$ cm. per second, while the velocity of violet light in the same glass is about $1 \cdot 96 \times 10^{10}$ cm. per second.

80. **Dispersion in a Lens.**—When a pencil of composite light is refracted through a lens dispersion takes place as in refraction through a prism, and the constituent pencils which emerge from the lens cannot, therefore, have a common focus.

Thus, in the case shown in Fig. 142, a pencil of white light diverging from a point, P, on the principal axis of the convex lens L, is refracted through the lens, and emerges as a convergent pencil. The pencil has, however, suffered dispersion by refraction through the lens, and the constituent rays of the emergent pencil do not all converge to a single focus conjugate to P. The violet rays of shortest wave-length are most refracted, and converge to a point, V, on the principal axis, while the red rays of longest wave-length are least refracted, and converge to a point, R, on the axis at a greater distance than V from the lens. Between these limits the intermediate rays of red, orange, yellow, green, blue, and violet converge to foci which extend continuously from R to V on the axis of the lens.

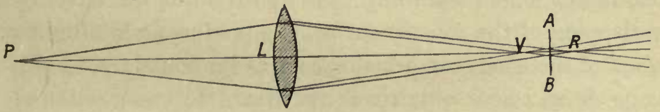

Fig. 142.

Hence, if a screen is placed, as at AB, to receive the image formed by the lens of the point P, it will be found that the image is not a very small, sharply-defined spot illuminated by white light, but a comparatively large circular white spot bordered by a coloured edge. It can be seen from the figure that, if the screen is nearer the lens than the position shown at AB, the outer edge of the image is coloured red, while if the screen is further from the lens than at AB, the outer edge of the image is coloured violet. When the screen is placed at AB the image is smaller and better defined than in any other position, and the coloured pencils converging to the foci between V and R overlap in such a manner that the coloured edge of the image practically disappears.

If the object at P is an object of finite size, and not a point, it will be clear from what has been said that the image will be a white image of the object bordered by a coloured fringe edged with red or violet, according to the position of the screen. The overlapping of the constituent pencils produces white light at all points of the emergent pencil except at its boundary, so that the image shows colour only round its edge. In one position of the screen the colour at the edge partially disappears, and the image is then best defined.

Experiment 75.—Set up a candle flame, a convex lens, and a screen in a dark room so as to obtain a real image of the flame on the screen.

It will be seen on examining the image that it is bordered by a well-defined coloured fringe, which is edged with red when the screen is too near the lens for good definition, and with blue or violet when the screen is too far from the lens.

It will be found, too, that the screen can be so adjusted in position that the colour at the edge of the image *almost* disappears. This is also the position in which the image is most clearly defined on the screen.

The dispersion which attends the refraction of white light through a lens, considered in relation to its effects on the colour and definition of the image formed by the lens, is generally known as the *chromatic aberration* of the lens.

It will be seen that, on account of the different refrangibilities of light of different colours, the focal length of a lens for any monochromatic light depends upon the colour of the light. The focal length evidently decreases as the refrangibility of the light increases, being least for violet light, and greatest for red light. The difference between the extreme values for violet light of shortest wave-length and red light of longest wave-length depends upon the material of the lens, but is, in all cases, very small. For example, if the focal length of a crown glass lens for violet light is about 150 mm., its focal length for red light would be about 152 mm. or 153 mm. Similarly, if a lens is used to give an image of any object with monochromatic light, the distance of the image from the lens for a given position of the object will depend upon the colour of the light, and will be slightly greater for red light than for yellow light, and greater for yellow light than for blue light.

A good lens, free from all defects of construction, will always give a clear, sharply-defined image of any object with monochromatic light, but if white or composite light is used, then the lens forms not one, but many, images of the object. Every constituent of the light gives a separate image of the object, and, as the images are all at slightly different distances from the lens, they cannot all be in focus at the same time on a screen. If the screen is placed so that any one image is in focus, then the others are all out of focus, and the composite image formed on the screen is badly defined, and is bordered with a coloured fringe, as explained above.

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